Quantum Information Science with Trapped Ions

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The Cambridge-MIT Partnership Programme

Summer School Belfast, 1 September 2005

UNIVERSITÄT SIEGEN

Overview

1. Ion Trap and Laser Cooling

2. Qubits and Quantum Gates

3. Ion Spin Molecules

4. QIS with trapped Yb^+ ions

Overview

1. Ion Trap and Laser Cooling
   - Electrodynamic trap
   - Collective ion motion: harmonic oscillator
   - Doppler cooling
   - Trapped atom-light interaction
   - Resolved sideband cooling.

A localised single atom

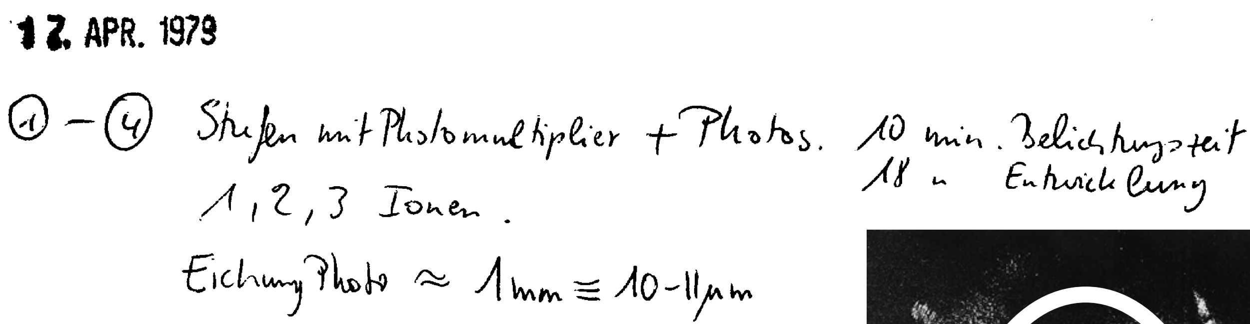
E. Schrödinger:

... we *never* experiment with just *one* electron or atom ...

... we are not *experimenting* with single particles, any more than we can raise Ichthyosauria in the zoo.

Br. J. Philos. Sci. III, August 1952.

W. Neuhauser *et al.*: single Barium ion

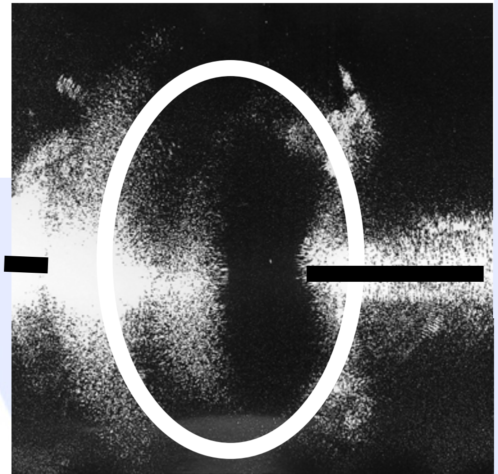

12. APR. 1979

① – ④ Stufen mit Photomultiplier + Photos. 10 min. Belichtungszeit
18 " Entwicklung

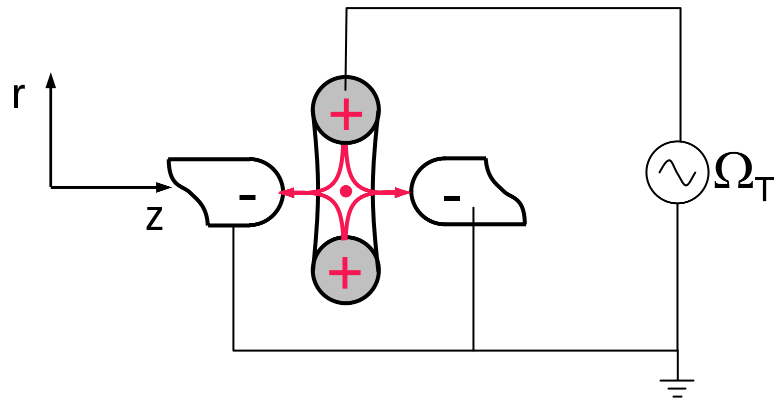
1, 2, 3 Ionen.

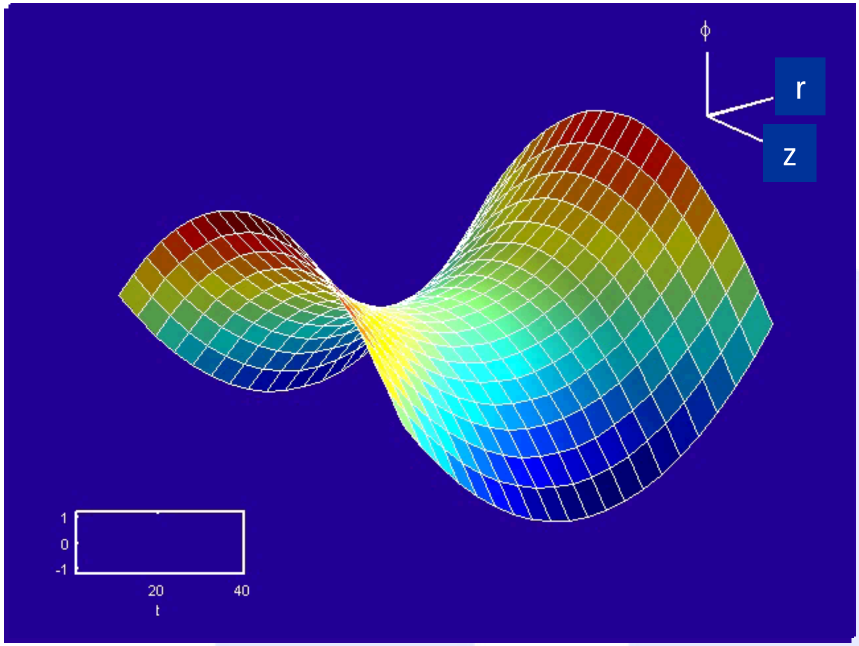
Eichung Photo ≈ 1 mm ≙ 10-11 μm

W. Neuhauser, M. Hohenstatt, P. E. Toschek, H.G. Dehmelt, Phys. Rev. A **22**, 1137 (1980).



Electrodynamic Trap





Quadrupole field

$$\Phi(r,z,t) = -V_0 \cos\Omega_T t \quad (r^2 - 2z^2)$$

Electrodynamic Trap

3-d Potential close to the centre of the trap:

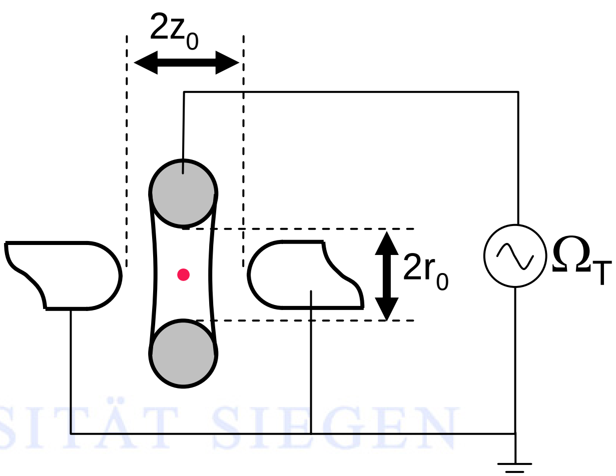
$$\Phi(r,z) = \frac{U + V\cos\Omega_t t}{r_0{}^2 + 2z_o^2}\left(x^2 + y^2 - 2z^2\right) \quad ,$$

Define $\quad a_z = -2a_{x,y} \equiv \dfrac{16\ eU}{m\Omega_T^2\left(r_o^2 + 2z_o^2\right)} \quad$ and $\quad q_z = -2q_{x,y} \equiv \dfrac{8eV}{m\Omega_T^2\left(r_0^2 + 2z_0^2\right)}$

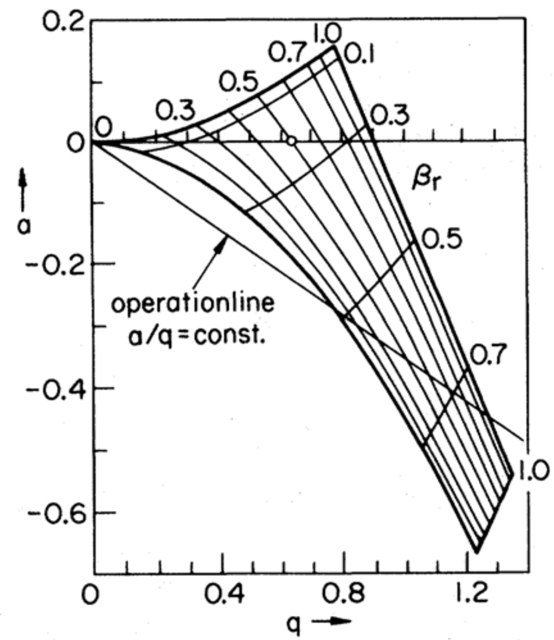
$$\tau \equiv \frac{\Omega_T t}{2}$$

Equations of motion (Mathieu equations) :

$$\frac{d^2 x_i}{d\tau^2} + \left(a_i - 2q_i\cos 2\tau\right)x_i = o \ , \quad i = x, y, z$$



Stable solutions:

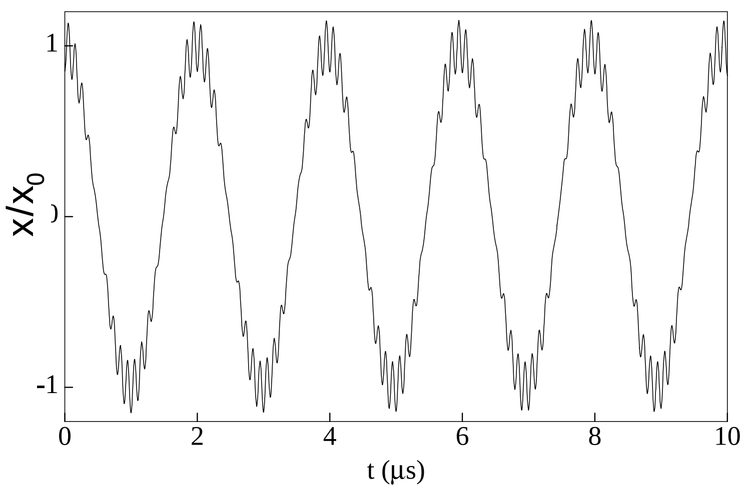


Electrodynamic Trap

Stable Solutions :

$$x_i(t) = x_0 \cos \nu_i t \left(1 - \frac{q_i}{2} \cos \Omega_T t\right)$$

with secular frequency $\nu_z = 2\nu_{x,y} = \frac{\Omega_T}{2}\sqrt{a_{x,y} + q_{x,y}^2/2}$ for $|a_i| \ll |q_i| \ll 1$



1-d harmonic motion weakly modulated with Ω_T. Here $\Omega_T = 9.5$ MHz , $q_r = 0.3$

for $|a_i|, \; |q_i| \ll 1$ effective potential:

$$V_{\text{eff}} = \frac{1}{2}\sum_i m\nu_i^2 x_i^2$$

Potential depth typically 10^2eV

for an overview (also Penning traps) see: P. K. Gosh, *Ion Traps* (Clarendon, Oxford, UK, 1995).

Harmonic Oscillator

Quantised motion:

$$\hat{x}_i = \sqrt{\frac{\hbar}{m\nu_i}}\left(a_i^\dagger + a_i\right) \quad , \qquad \hat{p}_i = \sqrt{\frac{m\hbar\nu_i}{2}}\left(a_i^\dagger - a_i\right)$$

Hamiltonian

$$H_{\text{ext}} = \sum_{i=x,y,z} \hbar\nu_i \left(a_i^\dagger a_i + \frac{1}{2}\right)$$

Single ion confined around the field-free trap centre.

Need to store N>1 ions for scalable QIP.

$N > 1$:

- add Coulomb potential
- expand total potential around equilibrium positions up to second order

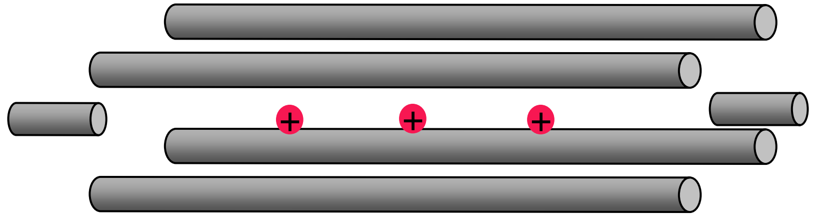
$\Rightarrow$ find $3N$ collective harmonic oscillator modes

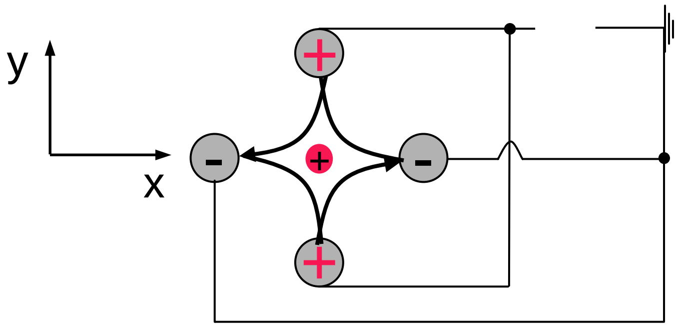
$$H_{\text{ext}} = \sum_i \sum_{j=1}^{N} \hbar\nu_{ij}\left(a_{ij}^\dagger a_{ij} + \frac{1}{2}\right)$$

Linear trap configuration: strong confinement in x- and y-direction, i.e., $\nu_{x,y} \gg \nu_z$

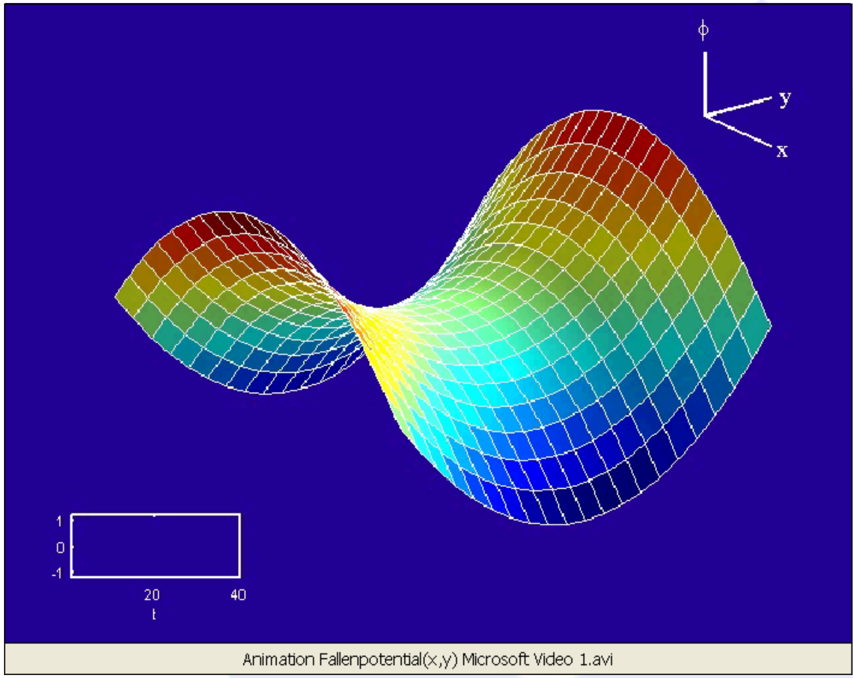
$\Rightarrow$ Consider axial modes only: $H_{\text{ext}} = \sum_{j=1}^{N} \hbar\nu_j\left(a_j^\dagger a_j + \frac{1}{2}\right)$

Electrodynamic trap



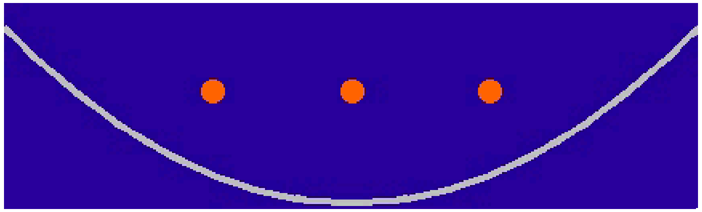


$$\Phi(x,y,t) = (U - V\cos\Omega_T t)\,\frac{x^2 - y^2}{2r_0^2}$$



Collective Harmonic Oscillator

N=3



$N > 1$:

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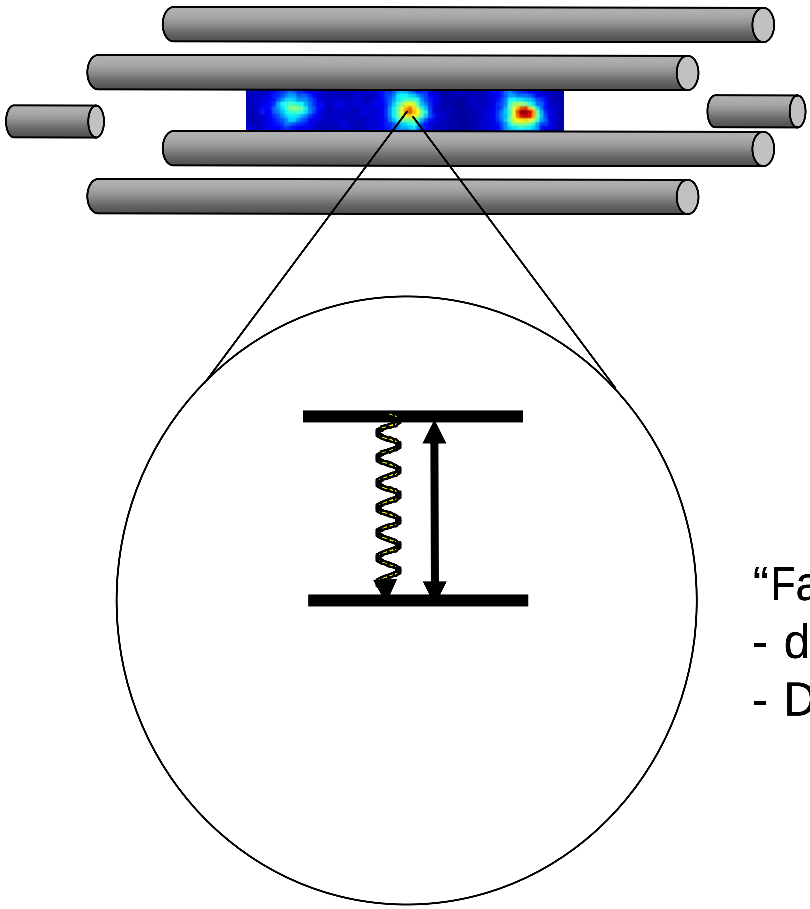
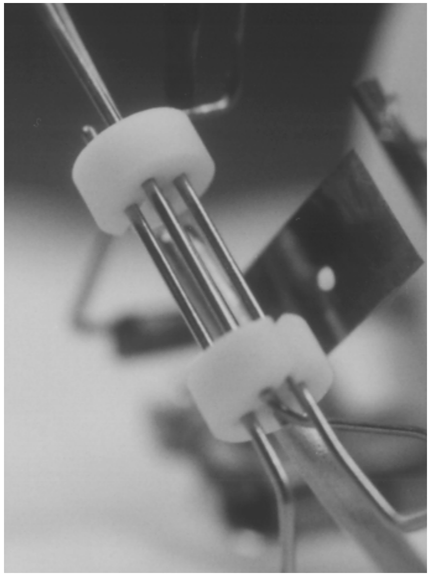
$\Rightarrow$ Consider axial modes only: $H_{\text{ext}} = \sum_{j=1}^{N} \hbar \nu_j \left(a_j^{\dagger} a_j + \frac{1}{2} \right)$

Detection of Trapped Ions

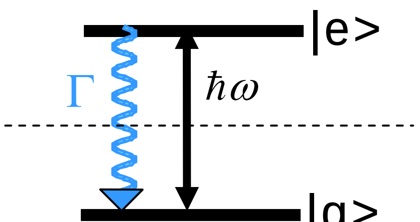
Yb^+ ion crystal





"Fast" (≈10MHz) dipole transition:
- detection of resonance fluorescence
- Doppler cooling.

Two-Level System



$$H_{\text{int}} = \tfrac{1}{2}\hbar\omega\sigma_z$$

Pauli matrices, spinor notation:

$$\tfrac{1}{2}\hbar\omega\left(|e\rangle\langle e| - |g\rangle\langle g|\right) \rightarrow \tfrac{1}{2}\hbar\omega\sigma_z = \tfrac{1}{2}\hbar\omega\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}$$

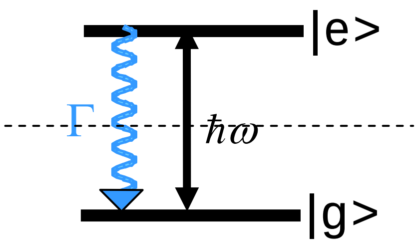
$$\left(|g\rangle\langle e| + |e\rangle\langle g|\right) \rightarrow \sigma_x = \begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}$$

$$i\left(|g\rangle\langle e| - |e\rangle\langle g|\right) \rightarrow \sigma_y = \begin{pmatrix}0 & -i\\ i & 0\end{pmatrix}$$

$$|g\rangle \rightarrow \begin{pmatrix}0\\ 1\end{pmatrix}$$

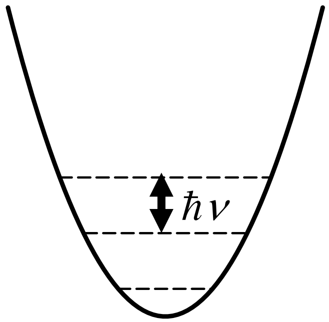
$$|e\rangle \rightarrow \begin{pmatrix}1\\ 0\end{pmatrix}$$

Doppler Cooling: $\Gamma \gg \nu$



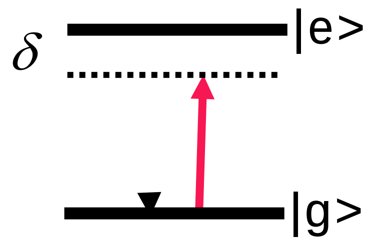
$$H_{\text{int}} = \tfrac{1}{2}\hbar\omega\sigma_z$$

$\otimes$

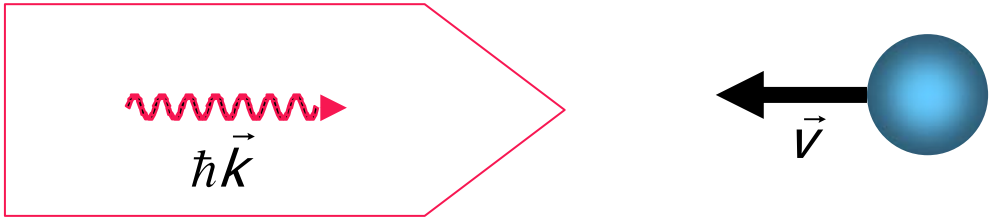


$$H_{\text{ext}} = \hbar\nu\left(a^{\dagger}a + \frac{1}{2}\right)$$

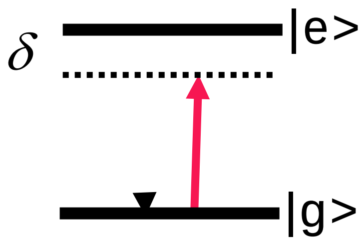
Doppler Cooling



resonant excitation for $\delta \simeq \vec{k} \bullet \vec{v}$

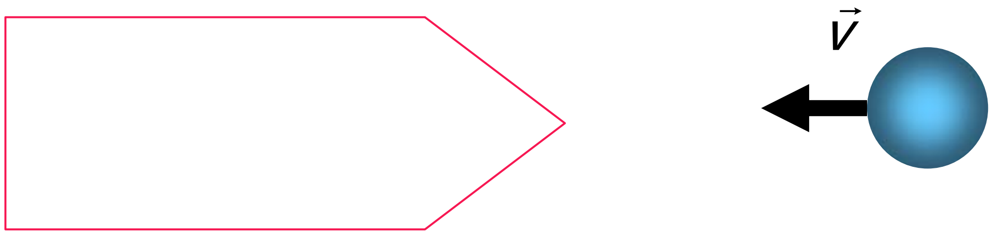


Doppler Cooling

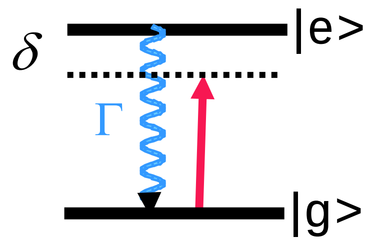


resonant excitation for $\delta \simeq \vec{k} \cdot \vec{v}$

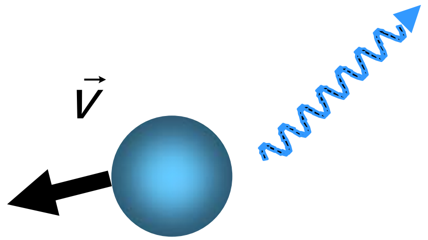
change of velocity $\Delta\vec{v} \simeq \hbar\vec{k} / m$



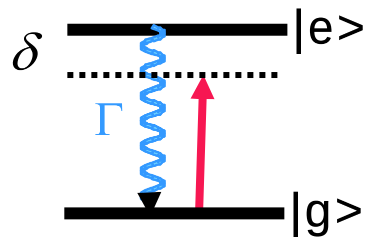
Doppler Cooling



spontaneous emission with rate Γ



Doppler Cooling $\Gamma \gg \nu$



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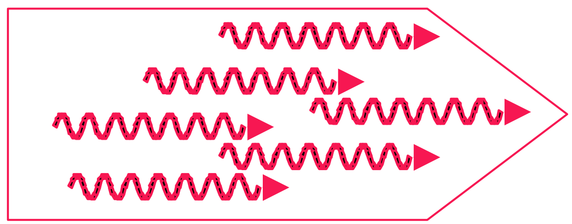
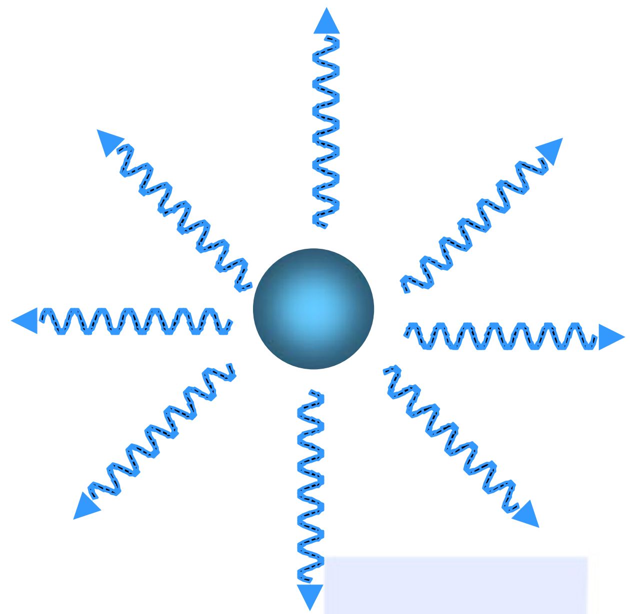


$l \times \hbar\vec{k}, \quad l \in \mathbb{N}$



Absorption: $\Delta\vec{p}_A = n \times \hbar\vec{k}$

Emission: $\langle \Delta\vec{p}_E \rangle = 0$

Diffusion in momentum space

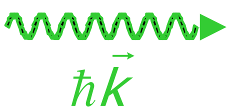
limits final temperature: $k_B T = \hbar\Gamma / 2$

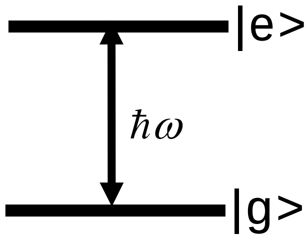
Ex.: $\nu = 1\text{MHz},\ \Gamma = 20\text{MHz} \ \Rightarrow \langle \text{n} \rangle_{\text{thermal}} \approx 10$

S. Stenholm, Rev.Mod. Phys. **58**, 699 (1986).

Trapped Atom-Light Interaction

2-level atom trapped in harmonic potential





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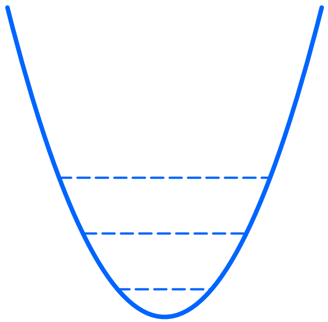

$$H_{int} = \tfrac{1}{2}\hbar\omega\sigma_z \qquad\qquad H_{ext} = \hbar\nu\left(a^\dagger a + \frac{1}{2}\right)$$

Interaction with near resonant lin.pol. travelling wave; lowest order in multipole expansion

$$H_L = \hbar\Omega_R\sigma_x \cos(kz - \omega_L t + \phi)$$

Rabi frequency $\Omega_R \equiv d_{eg}\cdot F_0/\hbar$

$$= \frac{1}{2}\hbar\Omega_R(\sigma_+ + \sigma_-)\left(e^{i(kz-\omega_L t+\phi)} + e^{-i(kz-\omega_L t+\phi)}\right)$$

With position operator $\hat{z} = \sqrt{\dfrac{\hbar}{2m\nu}}(a^\dagger + a) = \Delta z\,(a^\dagger + a)$

and Lamb-Dicke parameter $\eta \equiv \Delta z\, k = 2\pi\dfrac{\Delta z}{\lambda} = \sqrt{\dfrac{(\hbar k)^2}{2m}/\hbar\nu}$

$$\Rightarrow H_L = \frac{1}{2}\hbar\Omega_R(\sigma_+ + \sigma_-)\left(e^{i\left[\eta(a^\dagger + a) - \omega_L t + \phi\right]} + H.c.\right)$$

Trapped Atom-Light Interaction

Unitary transformation $\tilde{H}_L = e^{\frac{i}{\hbar}H_0 t} H_L e^{-\frac{i}{\hbar}H_0 t}$

with $H_o = H_{ext} + H_{\text{int}} = \hbar\nu\,(a^\dagger a + \frac{1}{2}) + \frac{1}{2}\hbar\omega\sigma_z$

$$\Rightarrow \tilde{H}_L = \frac{1}{2}\hbar\Omega_R\left[e^{i[(\omega-\omega_L)t+\phi]}\sigma_+ e^{i\eta\left[a^\dagger(t)+a(t)\right]} + H.c.\right]$$

where $a^\dagger(t) = a^\dagger e^{i\nu t}$ and $a(t) = ae^{-i\nu t}$

Expansion in η :

$$\tilde{H}_L = \frac{1}{2}\hbar\Omega_R\left[e^{i[(\omega-\omega_L)t+\phi]}\sigma_+\left[1 + i\eta(a^+ e^{i\nu t} + ae^{-i\nu t}) + ...\right] + H.c\right]$$

Lowest order in η :

$$\tilde{H}_L = \frac{1}{2}\hbar\Omega_R\left[e^{i[(\omega-\omega_L)t+\phi]}\sigma_+ + i\eta\left[e^{i(\omega-\omega_L+\nu)t}\sigma_+ a^+ + e^{i(\omega-\omega_L-\nu)t}\sigma_+ a\right] + H.c\right]$$

$\omega_L = \omega$, "Carrier"

$$\Rightarrow \tilde{H}_L = \frac{1}{2}\hbar\Omega_R\left(\sigma_+ e^{i\phi} + \sigma_- e^{-i\phi}\right)$$

$\omega_L = \omega - \nu$, $\phi = 0$, "red sideband"

$$\Rightarrow \tilde{H}_L = \frac{1}{2}\hbar\Omega_R\eta\left[\sigma_+ a + \sigma_- a^+\right]$$

Trapped Atom-Light Interaction

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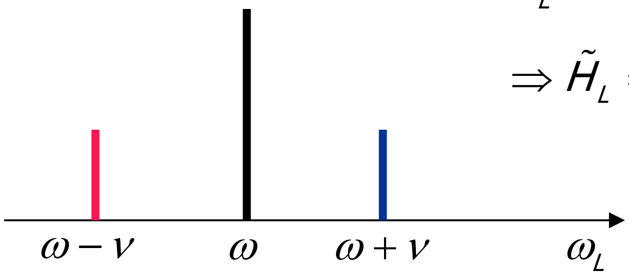
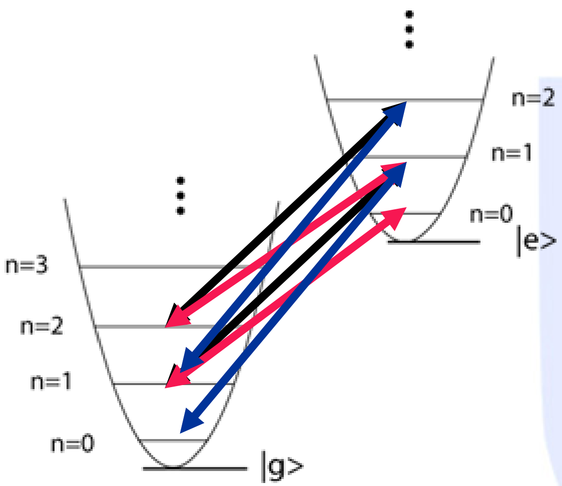
$$\Omega_{n,n-1} = \sqrt{n}\,\eta\,\Omega_R$$

$\omega_L = \omega + \nu,$ (ϕ=0) "blue sideband"

$$\Rightarrow \tilde{H}_L = \frac{1}{2}\hbar\Omega_R\eta\left[\sigma_+ a^\dagger + \sigma_- a\right]$$

$$\Omega_{n,n-1} = \sqrt{n+1}\,\eta\,\Omega_R$$





Trapped Atom-Light Interaction

$\omega_L = \omega,\ \phi = 0,$ "Carrier"

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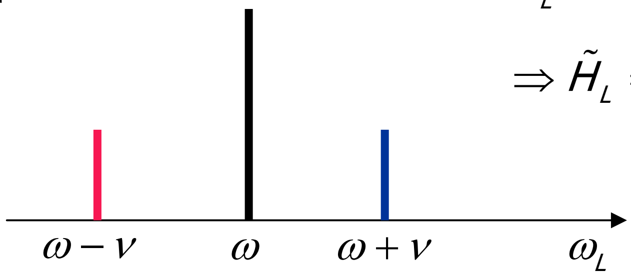
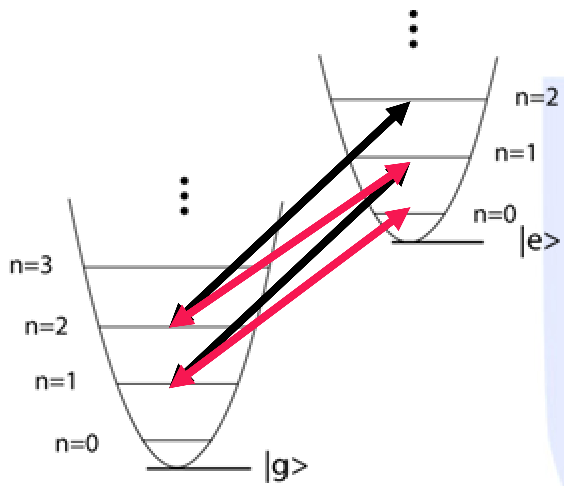
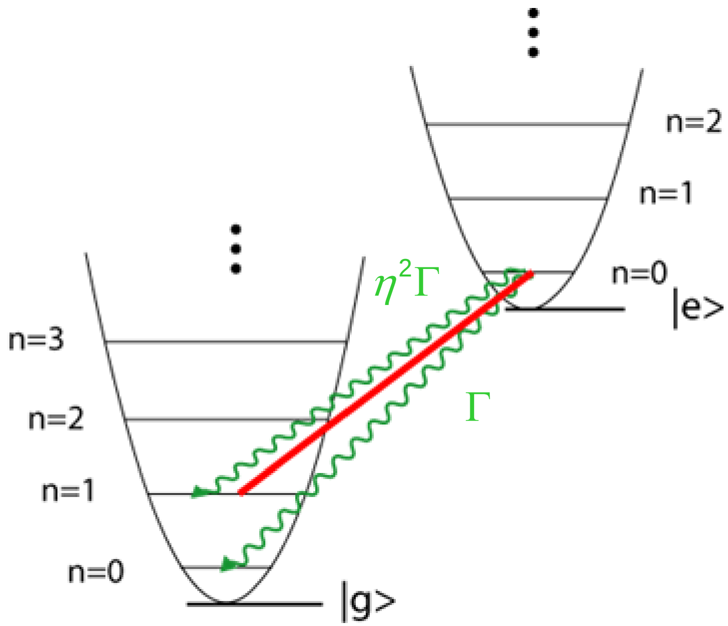
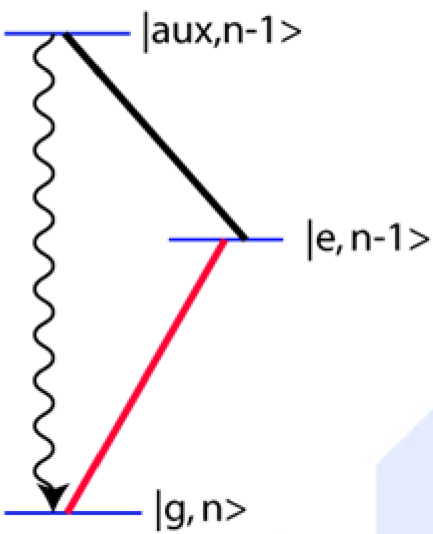




Resolved Sideband Cooling $\nu \gg \Gamma$





Take into account dissipation:

$$\frac{\partial \rho}{\partial t} = \frac{1}{i\hbar}[H,\rho] + \hat{L}\rho$$

Ground state cooling: $\langle n \rangle_{\text{thermal}} \approx 0$

for instance, F. Diedrich et al. PRL **62**, 403 (1989).

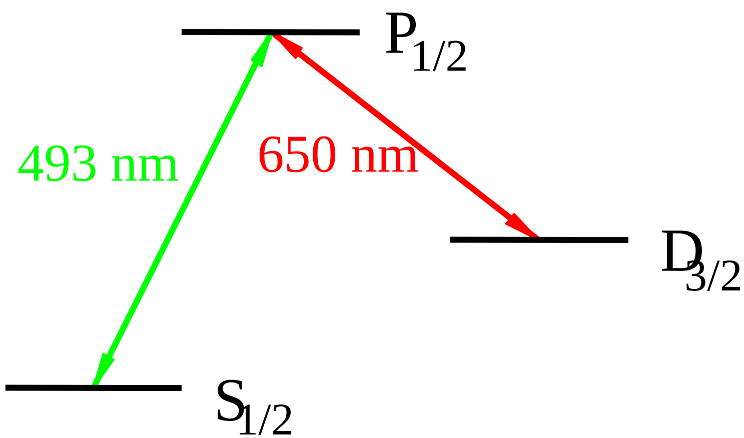
Cool collective vibrational modes

- Sequential sideband cooling of collective motion, e.g.:

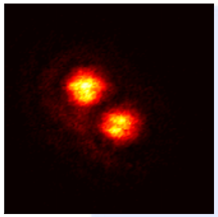
B. E. King et al. PRL **81**, 1525 (1998).
E. Peik et al. PRA **60**, 439 (1999).

- Shape atomic transitions by quantum interference, e.g.:

C.F. Roos et al. PRL **85**, 5547 (2000).
D. Reiß et al. PRA **65**, 053401 (2002).



Robust cooling of all modes well below the Doppler limit.
D. Reiß et al., PRA **65**, 053401 (2002)



- Simultaneous sideband cooling of many vibrational modes (theory):

CW, G. Morigi, D. Reiß, to appear in PRA.

Overview

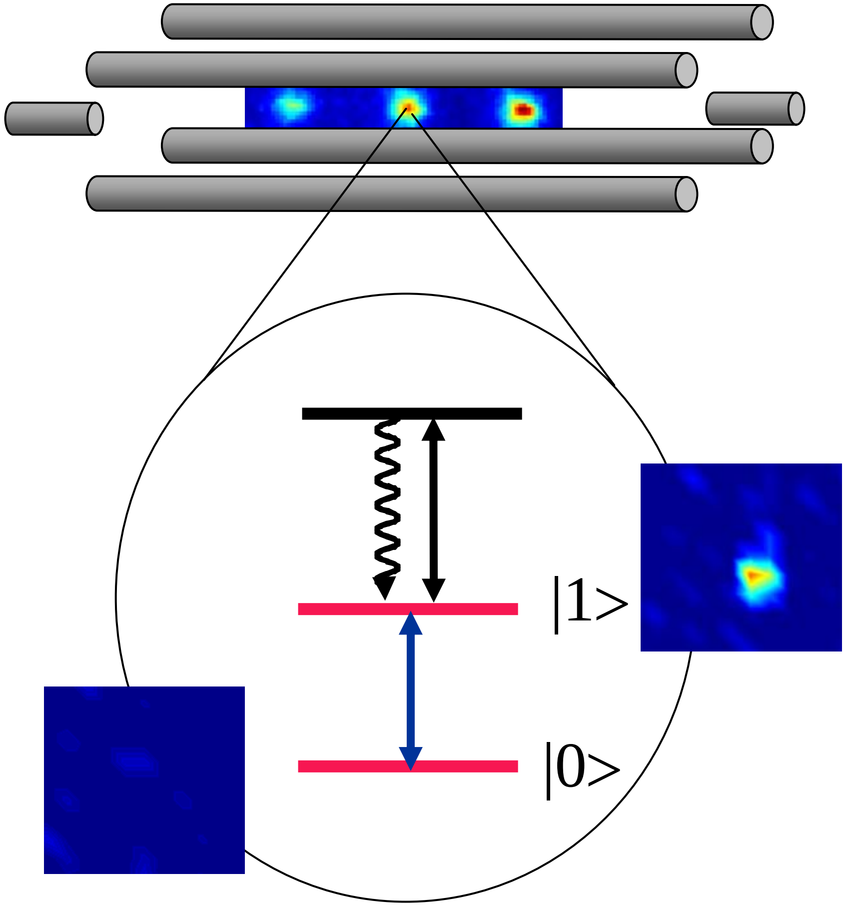
1. Ion Trap and Laser Cooling
   - Electrodynamic trap
   - Collective ion motion: harmonic oscillator
   - Doppler cooling
   - Trapped atom-light interaction
   - Resolved sideband cooling

2. Qubits and Quantum Gates
   - E2-transition, Hyperfine transition
   - Single qubit gates
   - 2-qubit gate

3. Ion Spin Molecules

4. QIS with trapped Yb^+ ions

Qubits: State Selective Detection

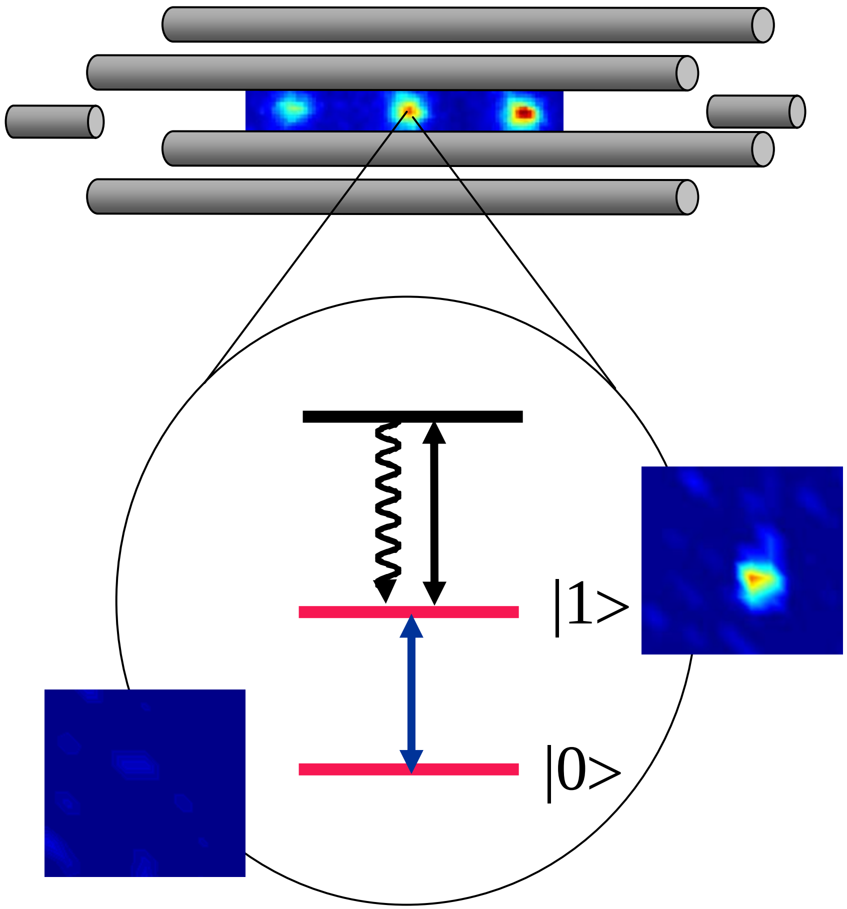


Internal states: qubits

Choose long-lived internal states as qubit.

State selective detection: detect resonance fluorescence; projective measurement of individual qubits.

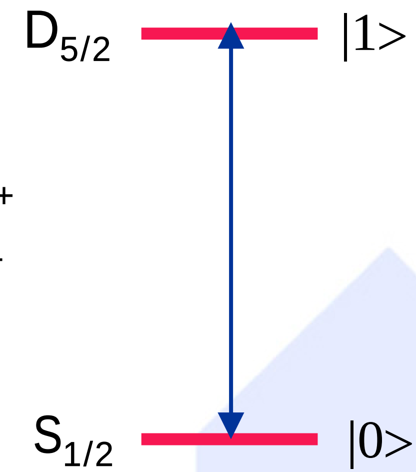
Qubits: E2 transition



Internal states: qubits

Electric quadrupole transition

Ba^+, Ca^+
Sr^+, Yb^+



Qubits: E2 transition

Interaction Hamiltonian

$$\tilde{H}_L = \frac{1}{2}\hbar\Omega_R\left(\sigma_+ e^{i\phi} + \sigma_- e^{-i\phi}\right)$$

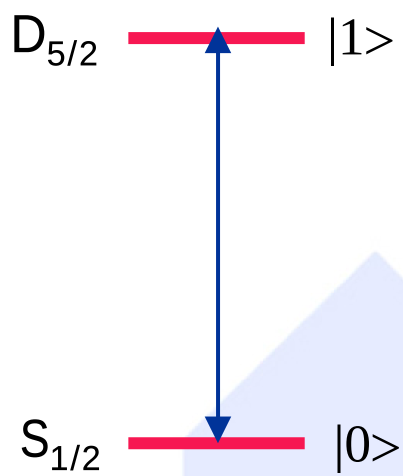
need to keep phase ϕ stable,

with $\omega \approx 5 \times 10^{14}$ Hz

$$\Rightarrow \Delta\omega / \omega \approx 10^{-13}$$

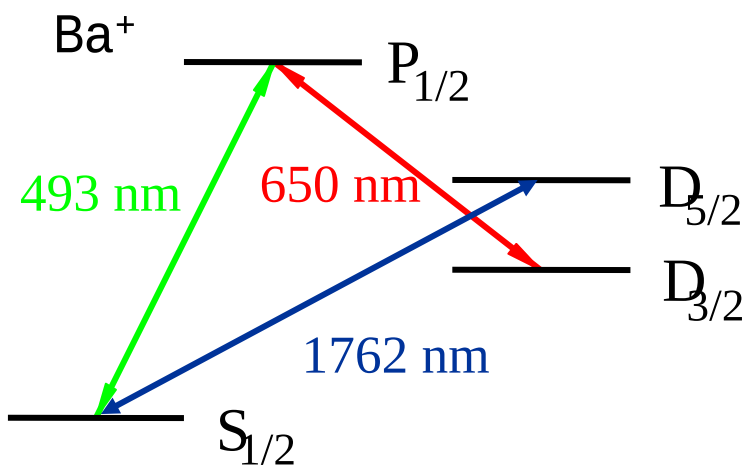
Electric quadrupole transition

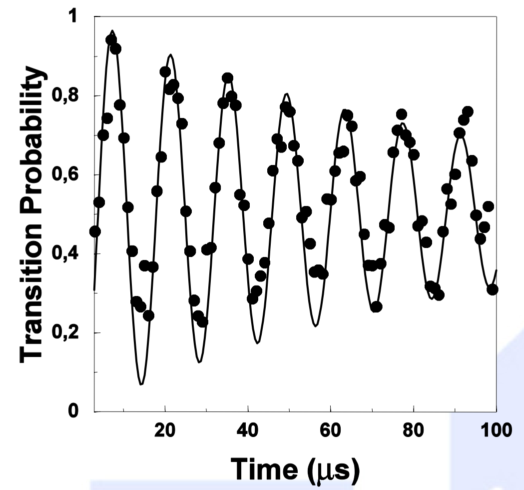
Ba^+, Ca^+
Sr^+, Yb^+



$\Rightarrow$ Precise coherent operations demand:
Small emission bandwidth, high absolute stability of frequency.

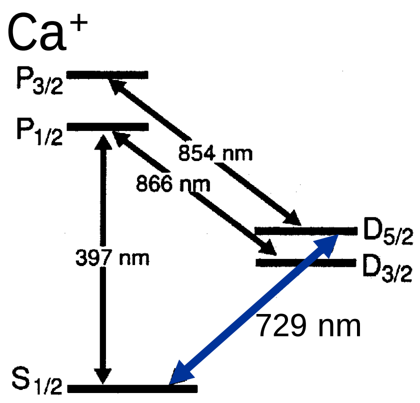
Qubits: E2 transition





- Coherent excitation of optical E2-transition
  CW, Chr. Balzer, Adv.At.Mol.Opt.Phys. **49**, 295 (2003).

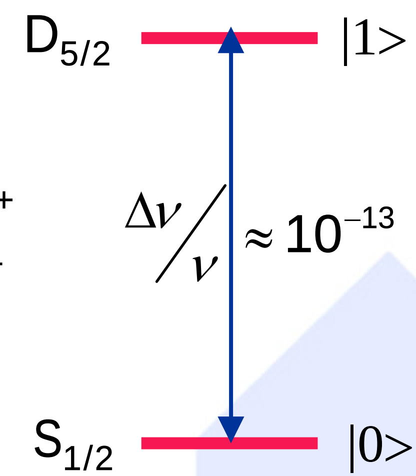
Qubits: E2 transition



e.g., Ch. Roos et al. PRL **83**, 4713 (1999)

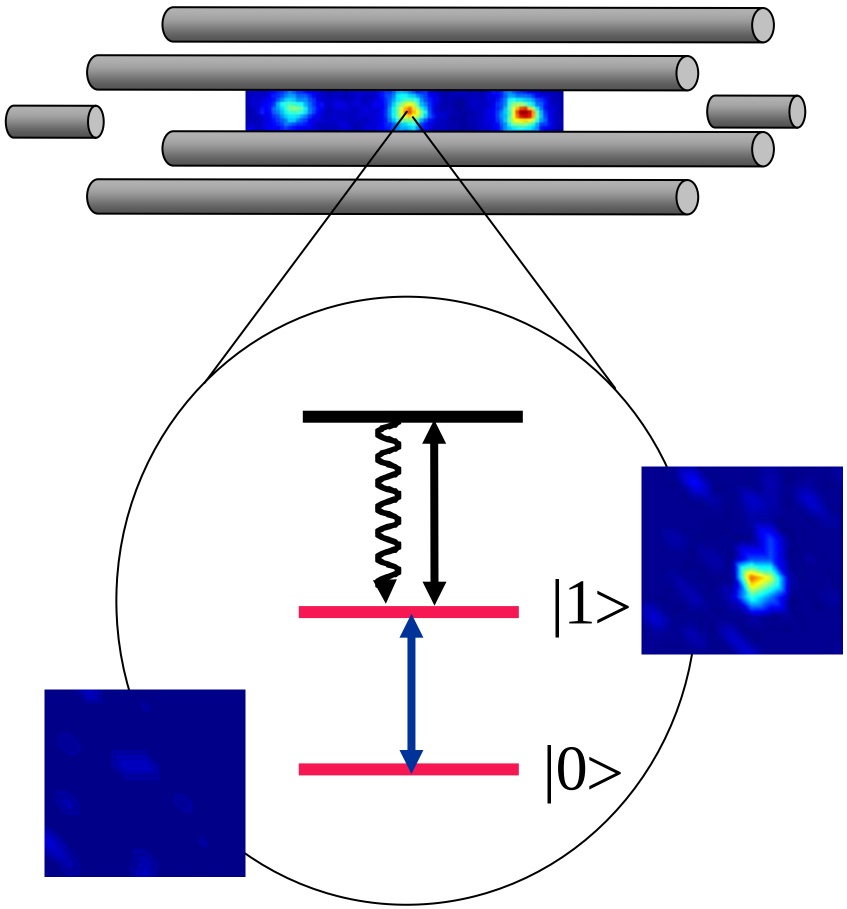
Electric quadrupole transition

Ba$^+$, Ca$^+$
Sr$^+$, Yb$^+$



$$\Delta\nu / \nu \approx 10^{-13}$$

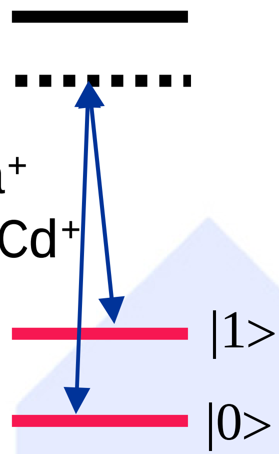
Qubits: Hyperfine transition



Internal states: qubits

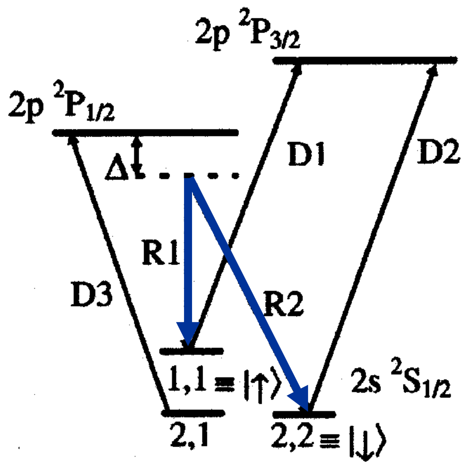
Hyperfine or Zeeman transition

$^{9}Be^{+}$, $^{25}Mg^{+}$, $^{43}Ca^{+}$
$^{87}Sr^{+}$, $^{137}Ba^{+}$, $^{115}Cd^{+}$
$^{171}Yb^{+}$



Qubits: Hyperfine transition

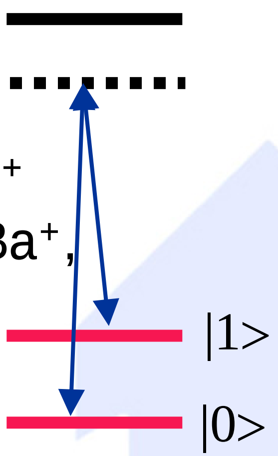
Be^+



e.g., C. Monroe et al.,PRL **75**, 4714 (1995).

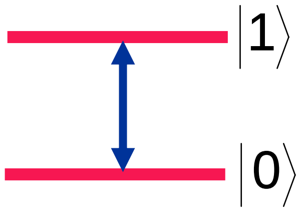
Hyperfine or Zeeman transition

$^9Be^+$, $^{25}Mg^+$, $^{43}Ca^+$, $^{87}Sr^+$, $^{111}Cd^+$, $^{137}Ba^+$, $^{171}Yb^+$



⇒ Precise coherent operations demand:
Small emission bandwidth, high absolute stability of frequency and intensity. Little off-resonant scattering.
Beam quality, pointing stability, diffraction.

Qubits



Qubit: $a|0\rangle + b|1\rangle$ where $|a|^2 + |b|^2 = 1$

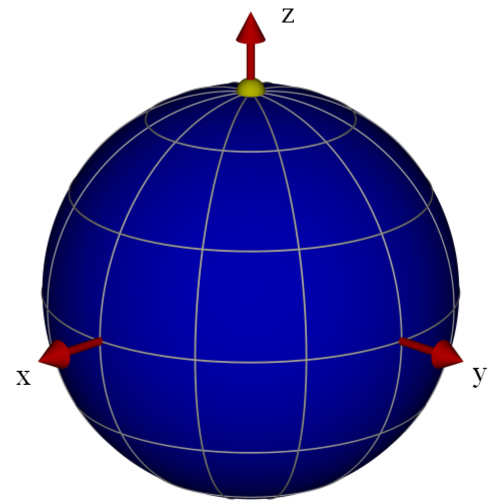
$$\cos\frac{\theta}{2}|0\rangle + \sin\frac{\theta}{2}e^{i\phi}|1\rangle \equiv |\theta,\phi\rangle$$

Quantum computing:

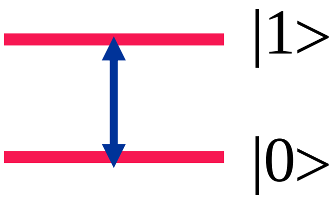
- Arbitrary single-qubit gates

- Conditional dynamics,
  e.g., CNOT gate

A. Barenco *et al.*, PRA **52**, 3457 (1995).



Single Qubit Gate



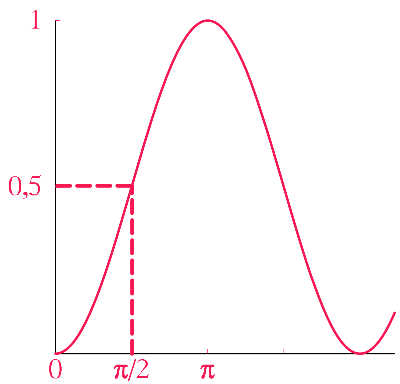
$$\omega_L = \omega$$

$$\Rightarrow \tilde{H}_L = \frac{1}{2}\hbar\Omega_R\left(\sigma_+ e^{i\phi} + \sigma_- e^{-i\phi}\right)$$

Time evolution operator (interaction picture) $U(t) = \exp\left(-\frac{i}{\hbar}\tilde{H}_L t\right)$

With $\phi = 0$: $\mathrm{U}(\vartheta) = \exp(-\mathrm{i}\frac{\vartheta}{2}\sigma_x) = \begin{pmatrix} \cos\vartheta/2 & -\mathrm{i}\sin\vartheta/2 \\ -\mathrm{i}\sin\vartheta/2 & \cos\vartheta/2 \end{pmatrix}$ where $\vartheta \equiv \Omega t$

Population in |1>



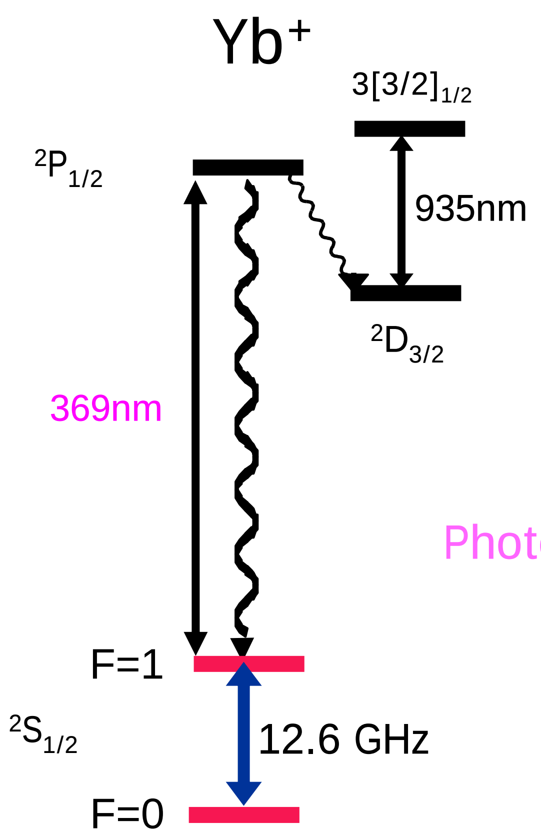
Nutation angle

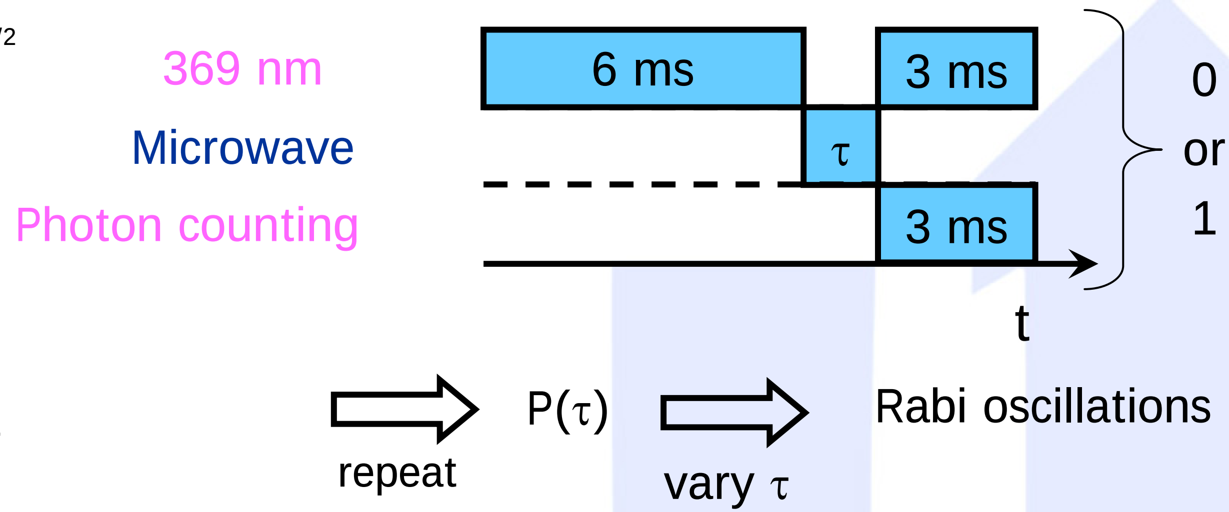
$$U\left(\vartheta = \tfrac{\pi}{2}\right)\begin{pmatrix}0\\1\end{pmatrix} = \begin{pmatrix}-i\sin\pi/4\\ \cos\pi/4\end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix}-i\\1\end{pmatrix} \mathrel{\hat{=}} \frac{1}{\sqrt{2}}\left(|0\rangle - i|1\rangle\right)$$

$$U\left(\vartheta = \pi\right)\begin{pmatrix}0\\1\end{pmatrix} = \begin{pmatrix}-i\sin\pi/2\\ \cos\pi/2\end{pmatrix} = \begin{pmatrix}-i\\0\end{pmatrix} \mathrel{\hat{=}} -i|1\rangle$$

$$U\left(\vartheta = 2\pi\right)\begin{pmatrix}0\\1\end{pmatrix} = \begin{pmatrix}-i\sin\pi\\ \cos\pi\end{pmatrix} = \begin{pmatrix}0\\-1\end{pmatrix} \mathrel{\hat{=}} -|0\rangle$$

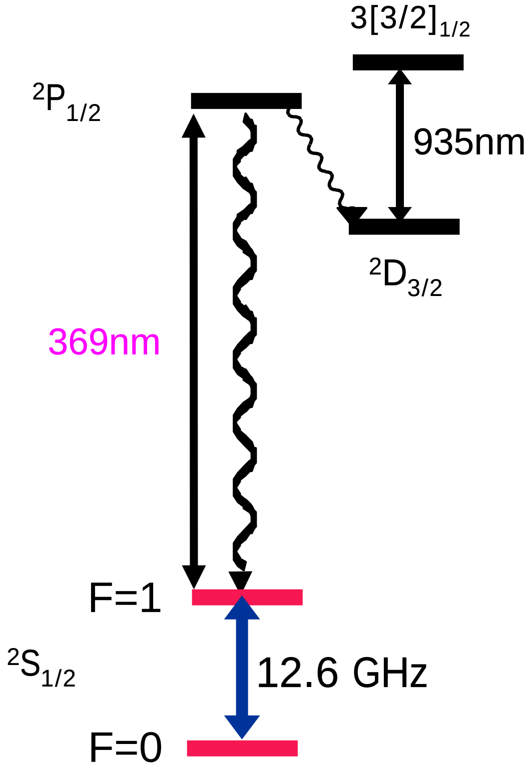
Single Qubit Gate



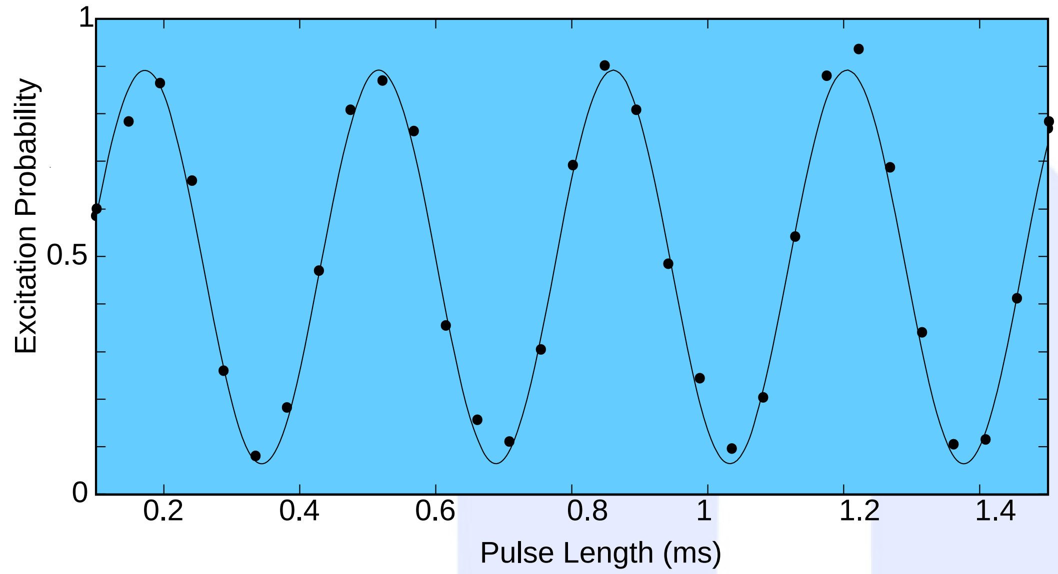


Single-Qubit Gate

Yb$^+$



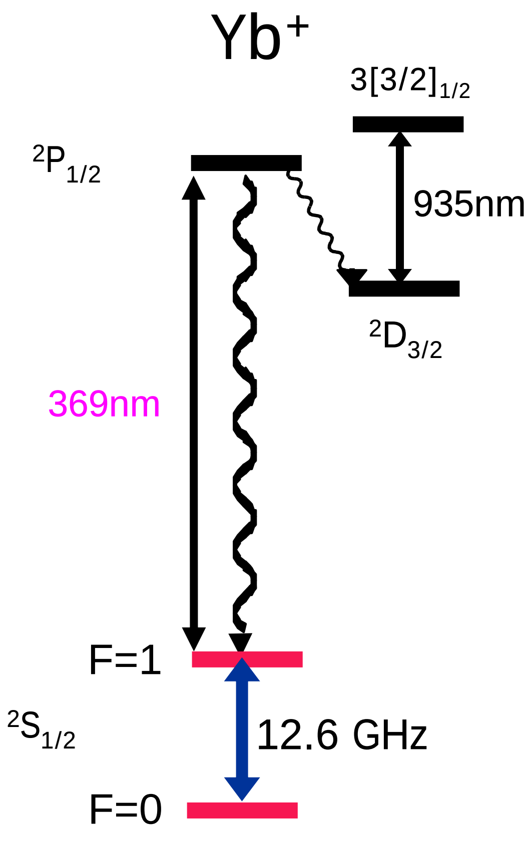
Individual Yb$^+$-Ion

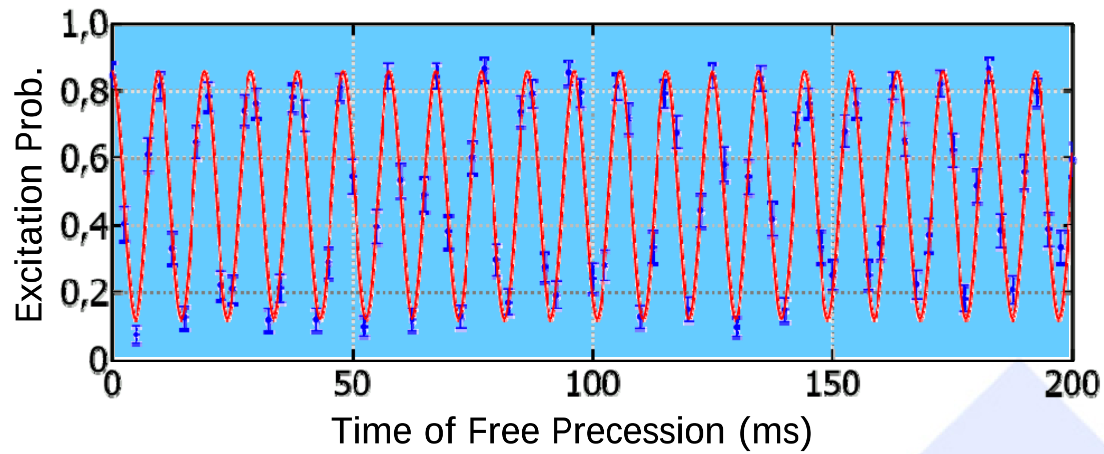


Rabi oscillations

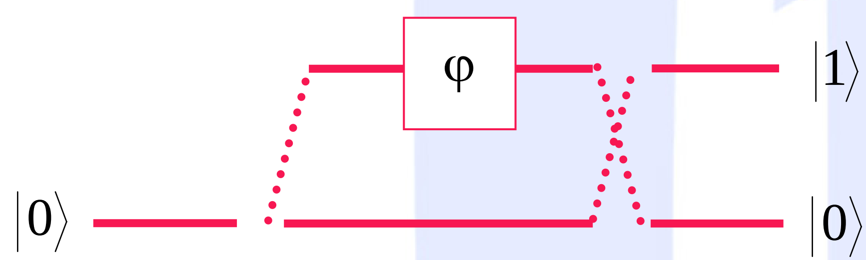
CW, Chr. Balzer, Adv.At.Mol.Opt.Phys. **49**, 295 (2003).

Single-Qubit Gate



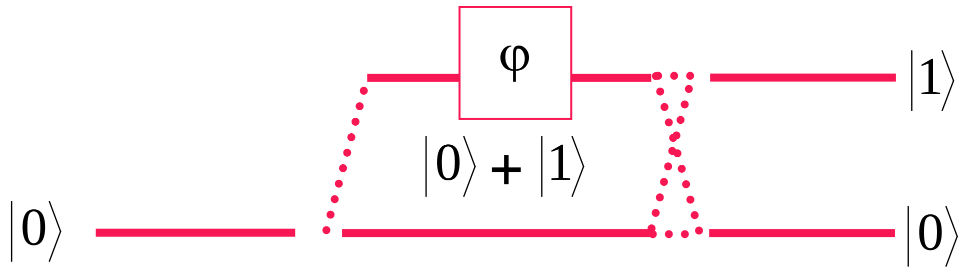


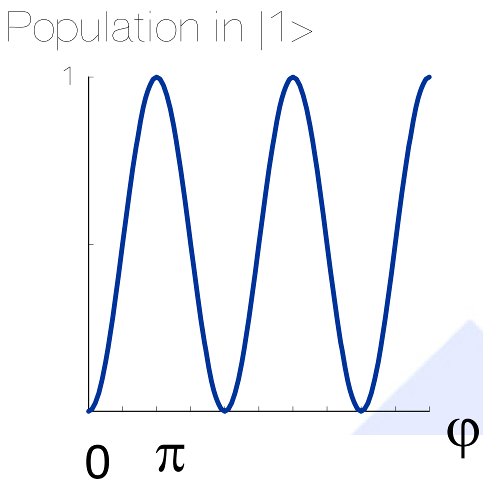
Single Atom Interferometer

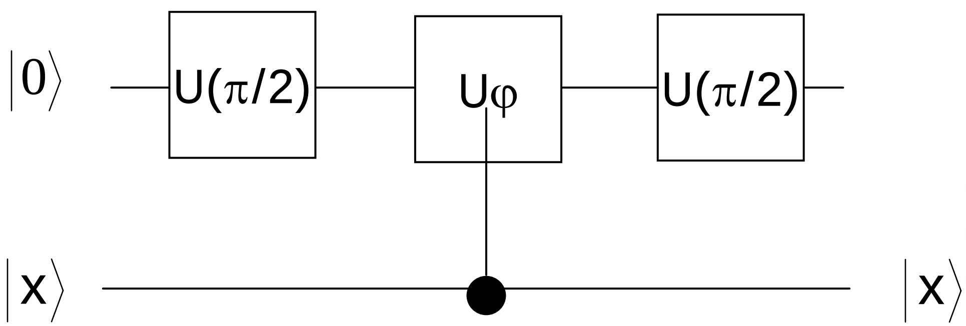


CW, Chr. Balzer, Adv.At.Mol.Opt.Phys. **49**, 295 (2003).

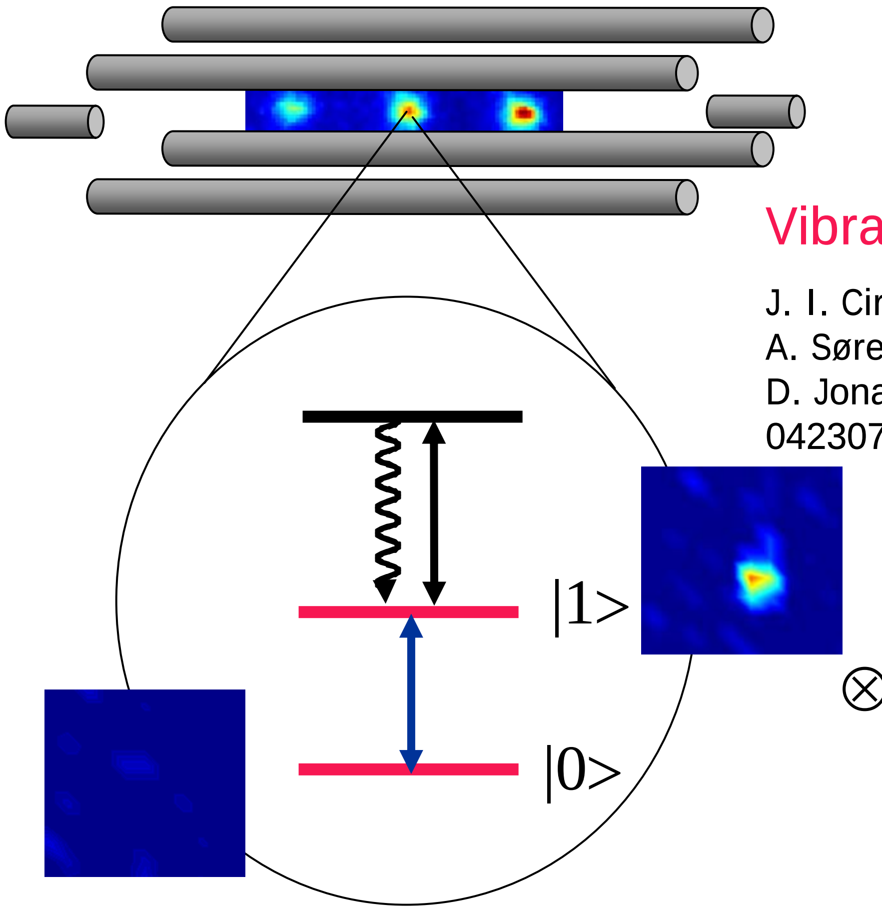
Two-Qubit Gate





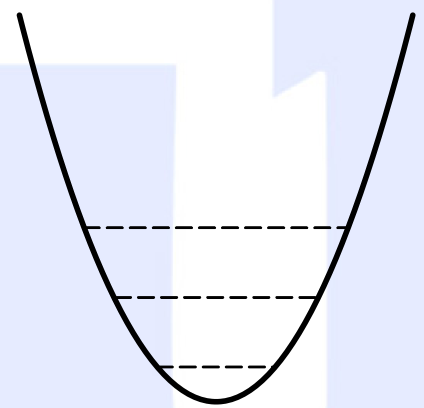


Two-Qubit Gate



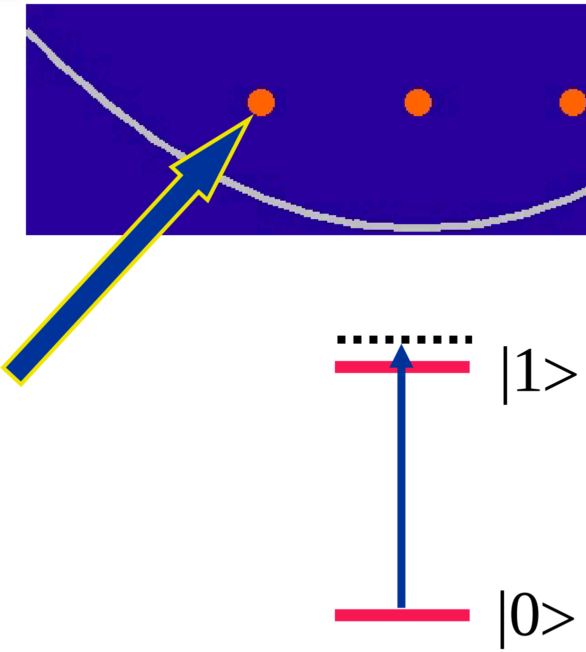
Vibrational motion: bus-qubit

J. I. Cirac, P. Zoller, PRL **74**, 4091 (1995)
A. Sørensen, K. Mølmer, PRA **62**, 022311 (2000)
D. Jonathan, M.B. Plenio, P.L. Knight, PRA **62**, 042307 (2000)



Internal states: qubits

Two-Qubit Gate



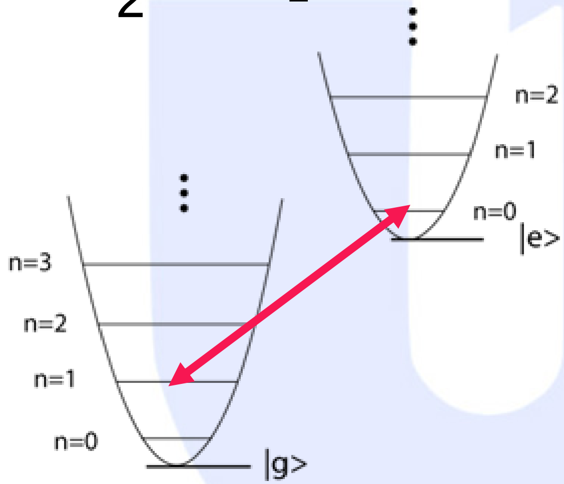
Electromagnetic radiation used to

- couple internal and external degrees of freedom

$$\eta \equiv \frac{\hbar k}{2p_0} = \frac{z_0}{\lambda} 2\pi$$

"Red sideband":

$$\tilde{H}_L = \frac{1}{2}\hbar\Omega_R\eta\left[\sigma_+ a + \sigma_- a^+\right]$$

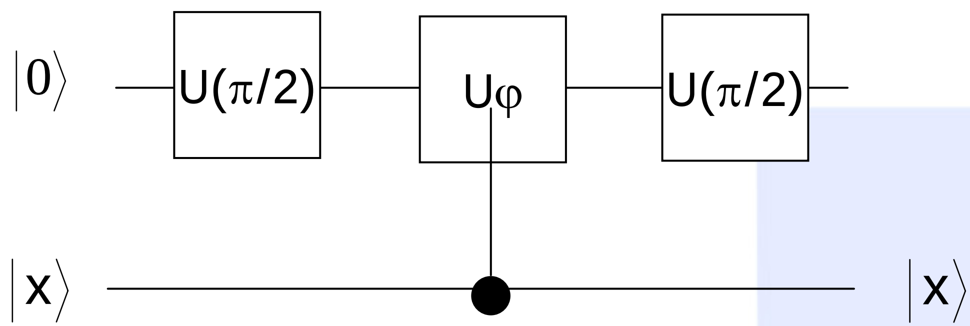


Two-Qubit Gate

CQ	TQ
0	0
0	1
1	0
1	1

CNOT →

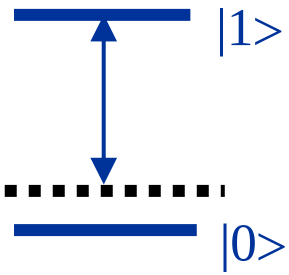
CQ	TQ
0	0
0	1
1	1
1	0



How to implement conditional phase shift?

Two-Qubit Cirac-Zoller Gate

π-pulse



CQ

TQ

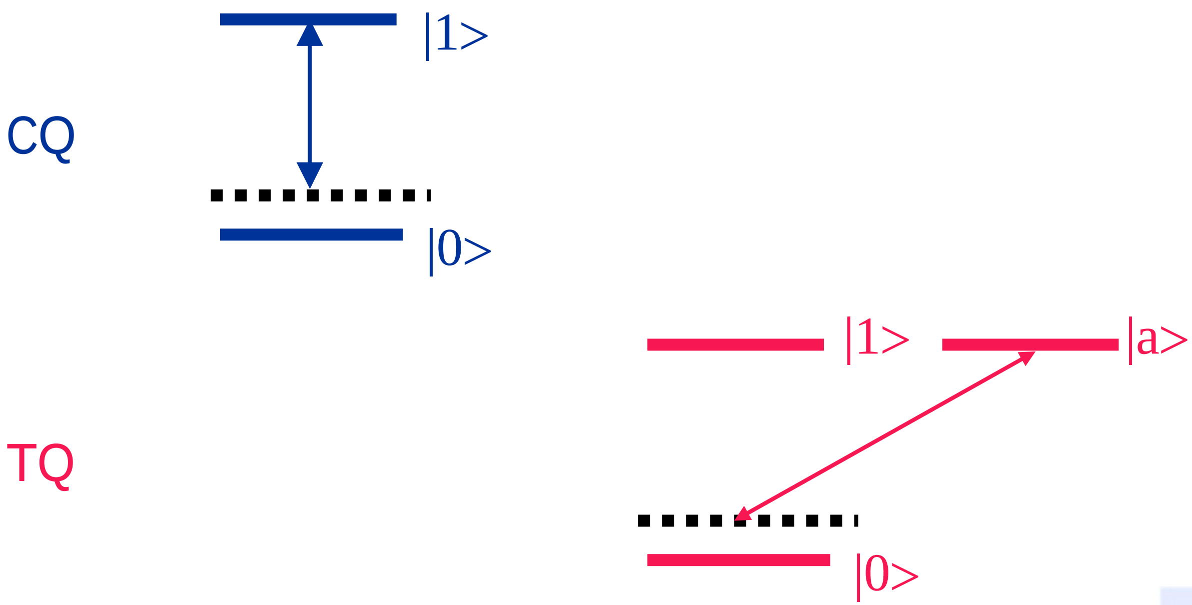
$$|1,0\rangle\,|0\rangle \rightarrow -i\,|0,0\rangle\,|1\rangle$$

$$|1,1\rangle\,|0\rangle \rightarrow -i\,|0,1\rangle\,|1\rangle$$

J. I. Cirac, P. Zoller, PRL **74**, 4091 (1995)

Two-Qubit Cirac-Zoller Gate

π-pulse → 2π-pulse

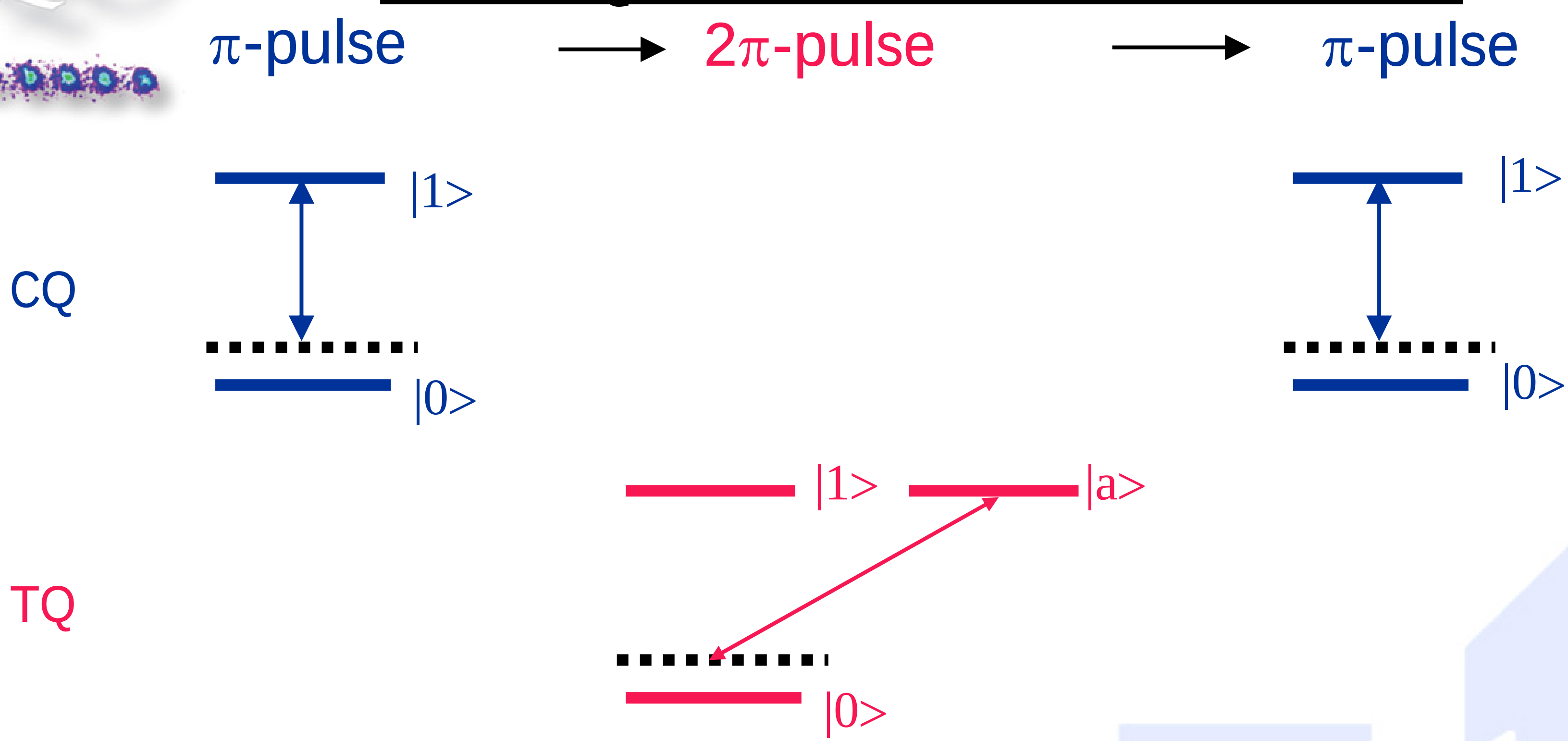


$$|1,0\rangle\,|0\rangle \rightarrow -i\,|0,0\rangle\,|1\rangle \rightarrow -(-i\,|0,0\rangle\,|1\rangle)$$

$$|1,1\rangle\,|0\rangle \rightarrow -i\,|0,1\rangle\,|1\rangle \rightarrow -i\,|0,1\rangle\,|1\rangle)$$

J. I. Cirac, P. Zoller, PRL **74**, 4091 (1995)

Two-Qubit Cirac-Zoller Gate



$$|1,0\rangle\,|0\rangle \rightarrow -i\,|0,0\rangle\,|1\rangle \rightarrow -(-i\,|0,0\rangle\,|1\rangle) \rightarrow -i(i\,|1,0\rangle\,|0\rangle) = |1,0\rangle\,|0\rangle$$

$$|1,1\rangle\,|0\rangle \rightarrow -i\,|0,1\rangle\,|1\rangle \rightarrow -i\,|0,1\rangle\,|1\rangle) \rightarrow -i(-i\,|1,1\rangle\,|0\rangle) = -\,|1,1\rangle\,|0\rangle$$

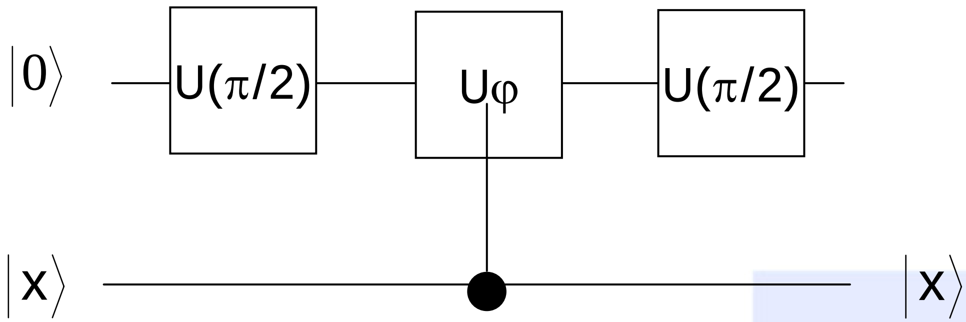
J. I. Cirac, P. Zoller, PRL **74**, 4091 (1995)

Two-Qubit Gate

CQ	TQ
0	0
0	1
1	0
1	1

CNOT →

CQ	TQ
0	0
0	1
1	1
1	0



Experiments, e.g.:

CNOT Internal state/Motion:

C. Monroe et al.,PRL **75**, 4714 (1995).

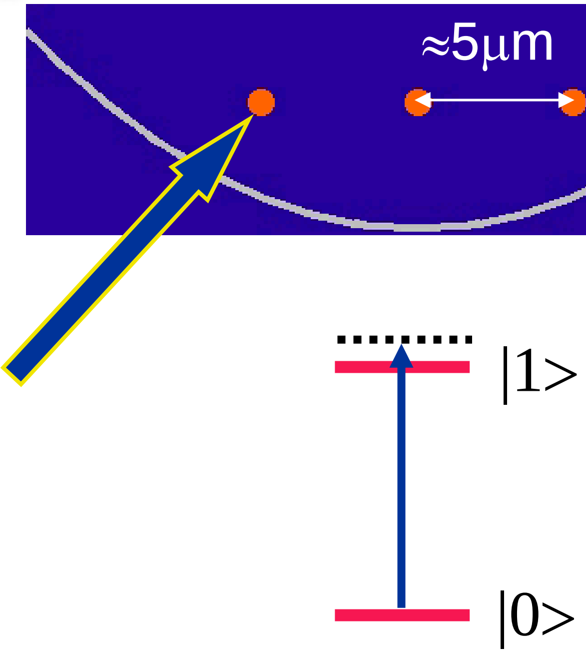
Cirac-Zoller Gate:

F. Schmidt-Kaler et al. Nature **422**, 408 (2003);

Geometric Phase Gate:

D. Leibfried et al. Nature **422**, 412 (2003).

QIP with trapped ions



Electromagnetic radiation used to

- couple internal and external degrees of freedom

$$\eta \equiv \frac{\hbar k}{2p_0} = \frac{z_0}{\lambda}2\pi \qquad z_0 \approx 10\,\text{nm}$$

$$H_I \propto \sigma_+ \exp\left[i\eta\left(a + a^+\right)\right] + \text{h.c.}$$

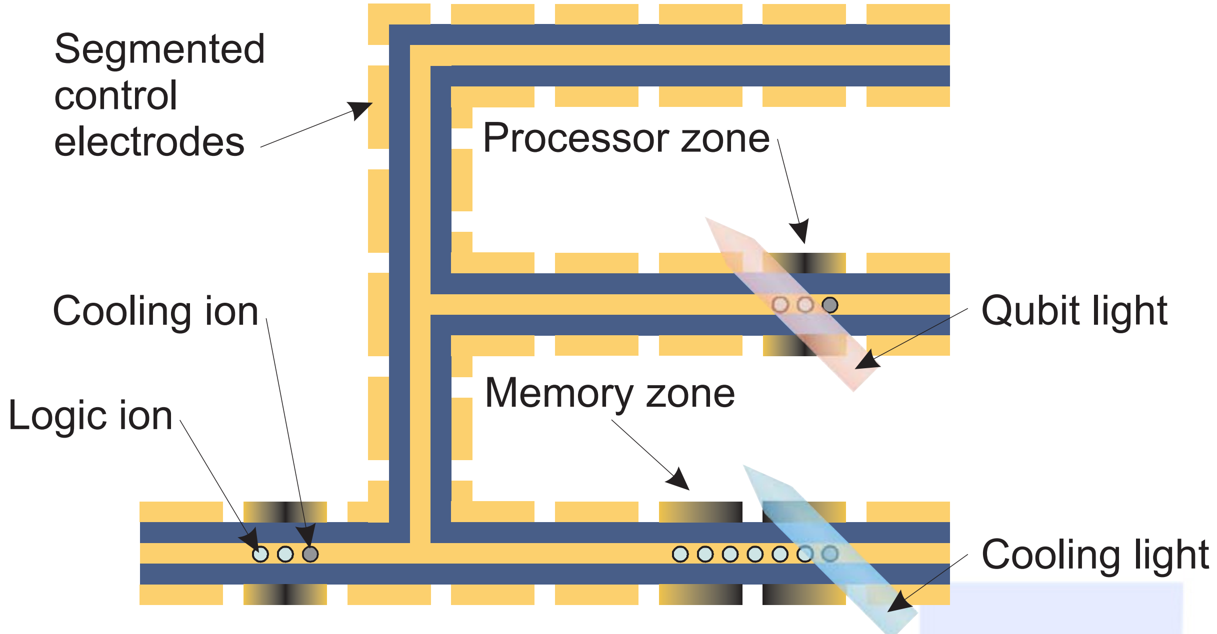
$\Rightarrow$ optical wavelengths

- address individual qubits

$\Rightarrow$ optical wavelengths

$\Rightarrow$ Precise coherent operations demand:
Small emission bandwidth, high absolute stability of frequency and intensity. Little off-resonant scattering.
Beam quality, pointing stability, diffraction.

Scaling



J. Chiaverini et al., Quant. Inf. Comput. **5**, 419 (2005).
D. J. Wineland et al., J. Res. Natl. Inst. Stand. Technol. **103** (3), 259 (1998).

Two ions at a time for quantum logic. Separate memory regions.
$\Rightarrow$ avoid cooling of many vibrational modes.
$\Rightarrow$ avoid individual addressing.

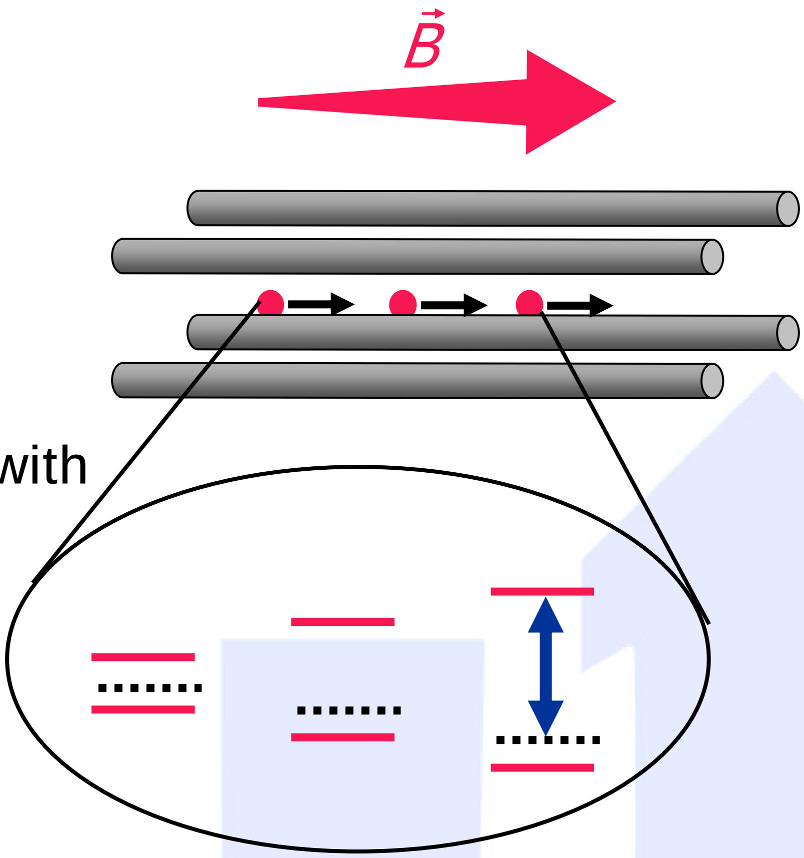
Overview

1. Ion Trap and Laser Cooling
   - Electrodynamic trap
   - Collective ion motion: harmonic oscillator
   - Doppler cooling
   - Trapped atom-light interaction
   - Resolved sideband cooling
2. Qubits and Quantum Gates
   - E2-transition, Hyperfine transition
   - Single qubit gates
   - 2-qubit gate
3. Ion Spin Molecules
   - Spin-Motion coupling
   - Spin-Spin coupling
   - Analogy with NMR
4. QIS with trapped Yb^+ ions

Spin resonance with trapped ions

State-dependent force:

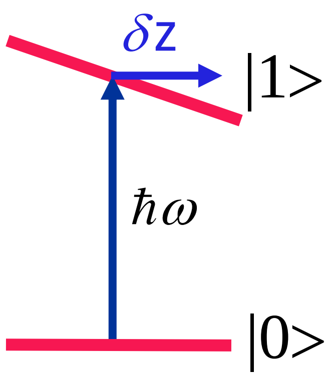
- Qubit resonances shifted individually
- Coupling of internal and external dynamics even with microwave radiation

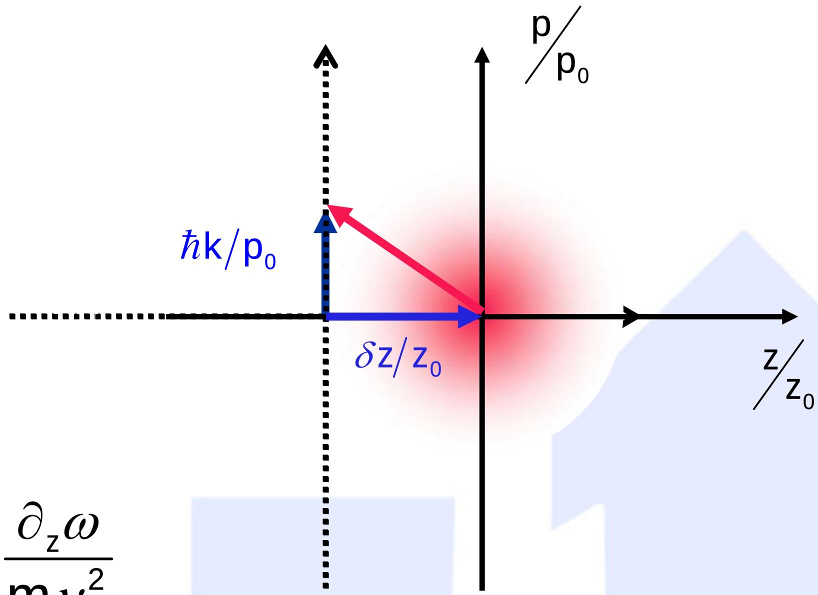


F. Mintert, CW, PRL **87**, 257904 (2001).

Spin-Motion Coupling

- Coupling internal and external dynamics: using state dependent force





Equilibrium shifted by $\delta z = -\hbar \frac{\partial_z \omega}{m\nu^2}$

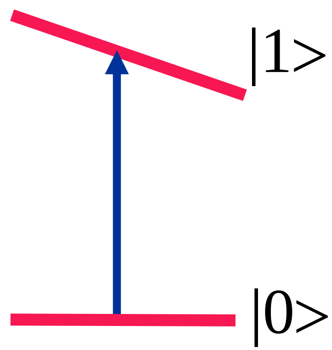
Coupling parameter: $\kappa \equiv \frac{\delta z}{z_0} = z_0 \frac{\partial_z \omega}{\nu}$

effective Lamb-Dicke parameter: $\eta' \equiv \eta - i\kappa$

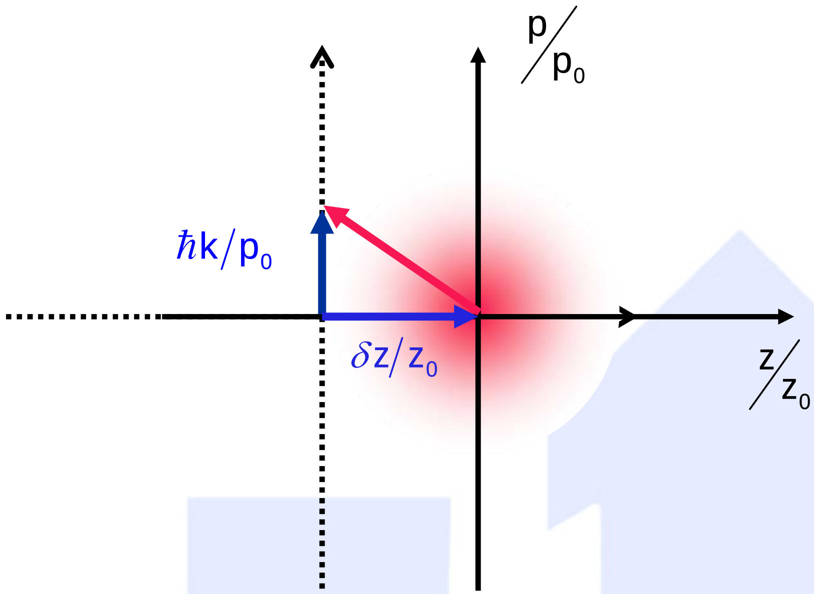
F. Mintert, CW, PRL **87**, 257904 (2001).

Spin-Motion Coupling

- Coupling internal and external dynamics: using state dependent force



$\otimes$



$H_I \propto \sigma_+ \exp\left[i\eta'\left(a + a^+\right)\right] + \text{ h.c.}$ where $\eta' \equiv (\eta^2 + \kappa^2)^{1/2}$

$\Rightarrow$ All optical schemes can be used with rf or mw radiation.

$\Rightarrow$ Applicable to neutral atoms, too.

F. Mintert, CW, PRL **87**, 257904 (2001).

Spin-Motion Coupling

Make use of state dependent *optical dipole force* for quantum gates, for instance:

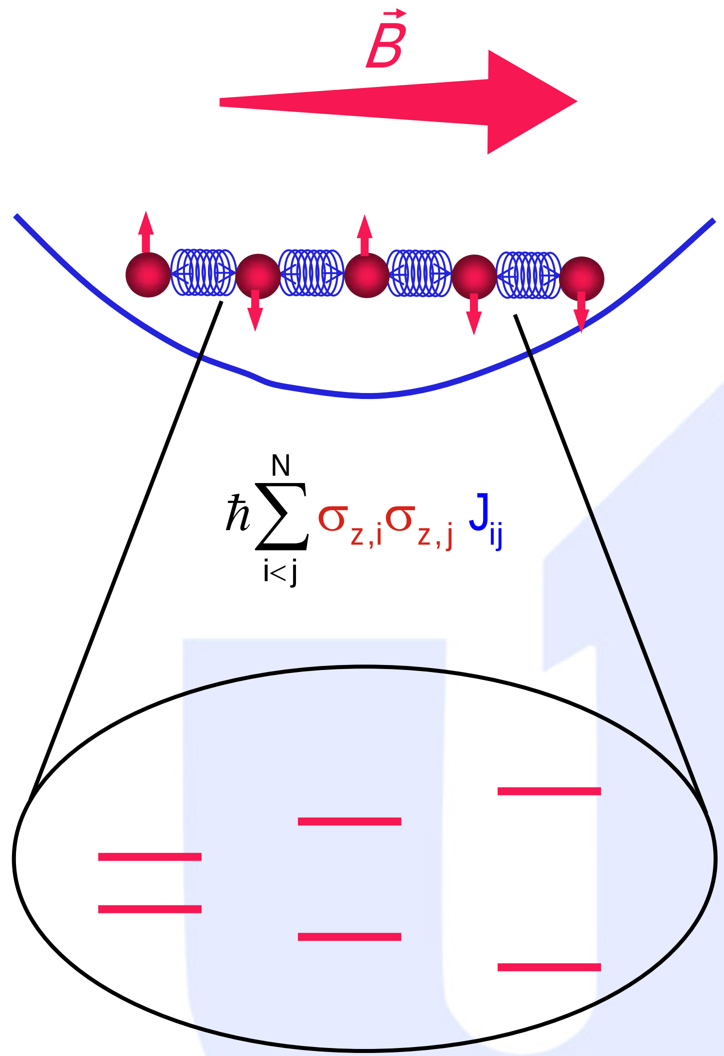
D. Leibfried et al., Nature **422**, 412 (2003). (Experiment)
D. Porras, J. I. Cirac, **92**, 207901 (2004). (Theory)
P. C. Haljan et al., PRL **94,** 153602 (2005). (Experiment)

Speed optimised quantum gates:
J. J. Garcıa-Ripoll, P. Zoller,J.I. Cirac, PRL **91**, 157901 (2003).

Spin resonance with trapped ions

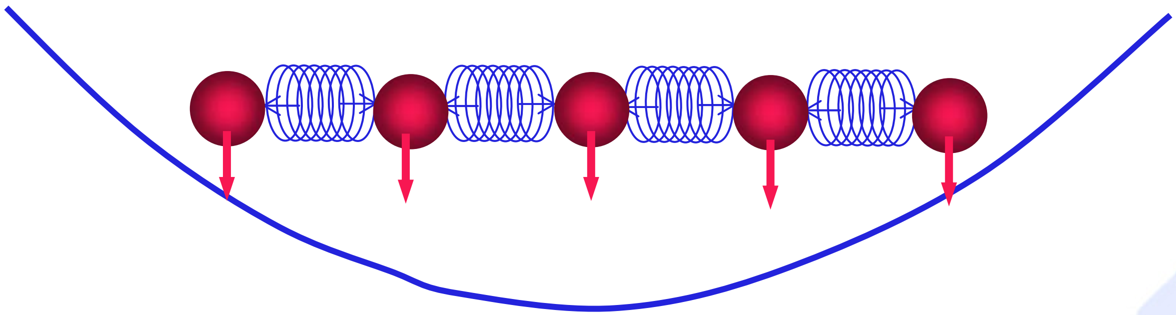
Ion Spin Molecule

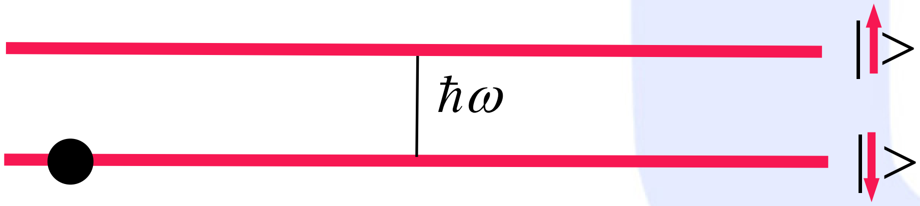
- Qubit resonances shifted individually
- Spin-Spin coupling between individual qubits



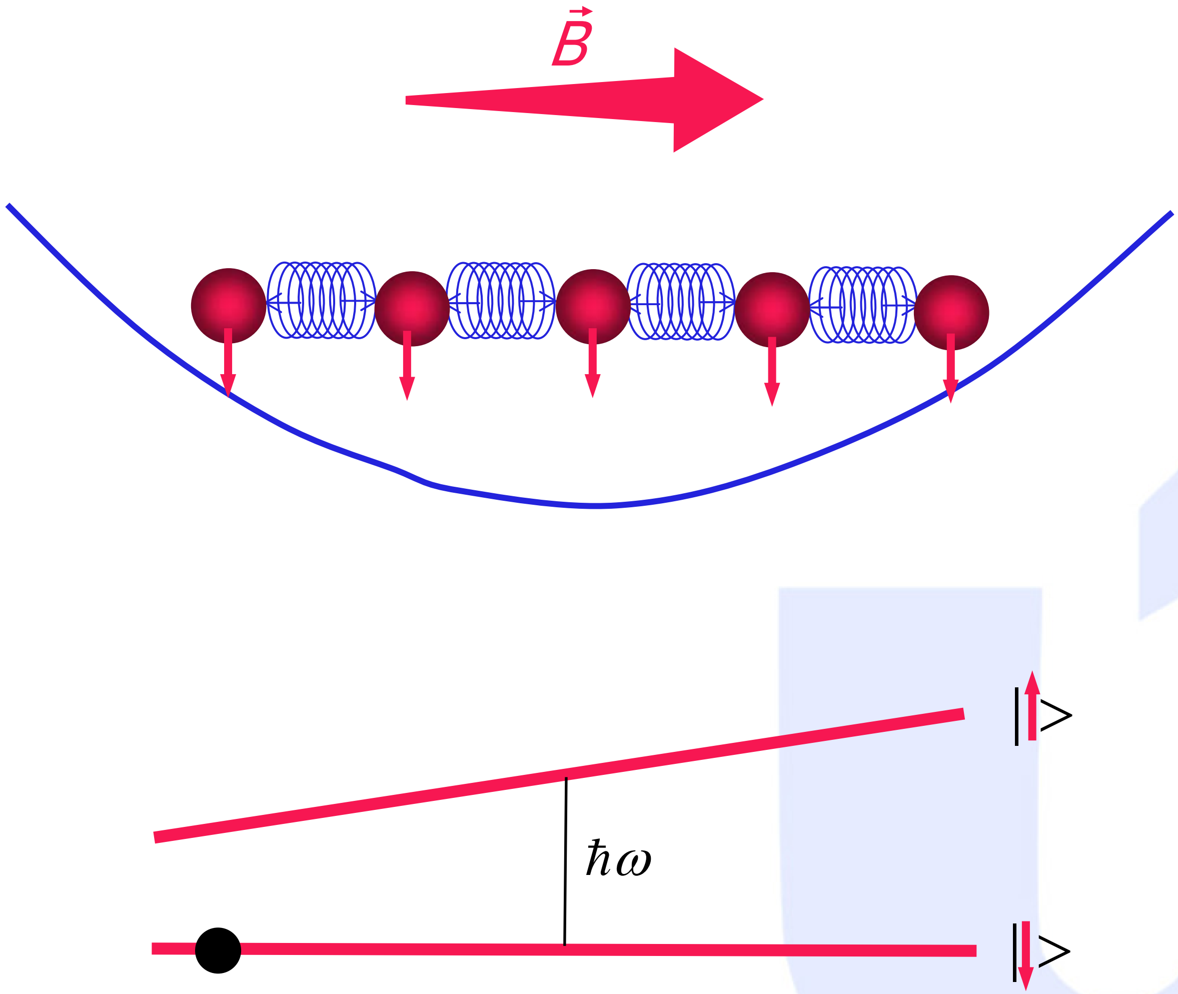
CW in *Laser Physics at the Limit,* Springer, 2002, p. 261.

Spin-Spin Coupling

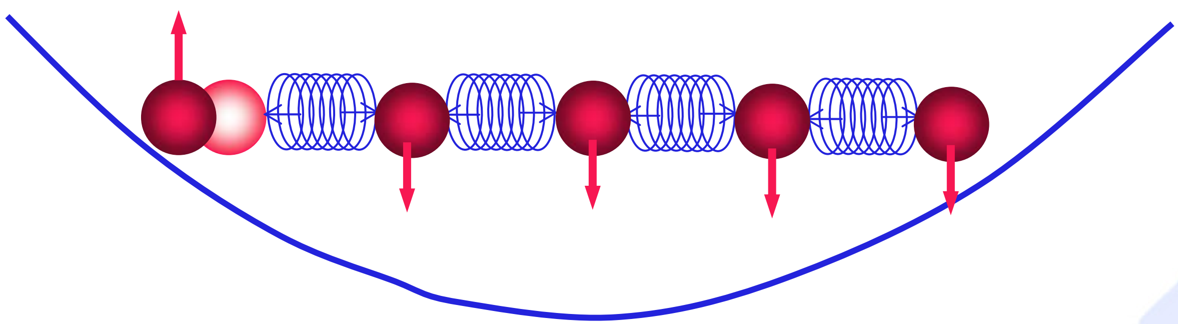


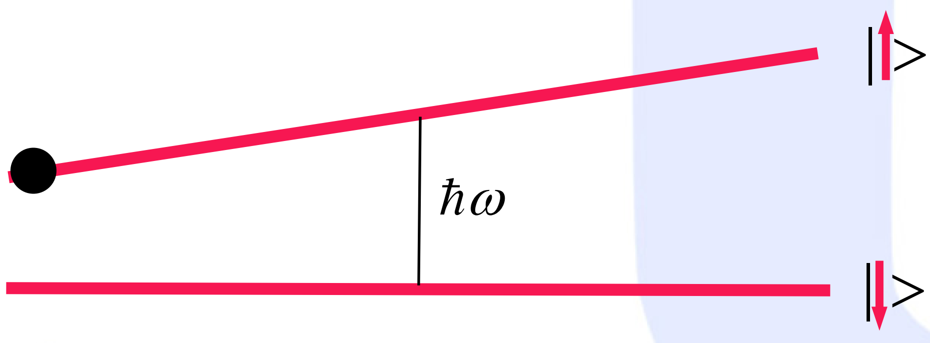


Spin-Spin Coupling

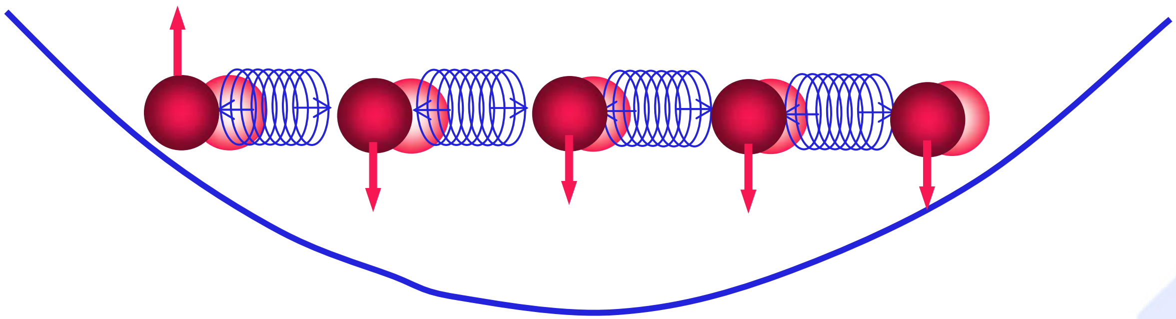


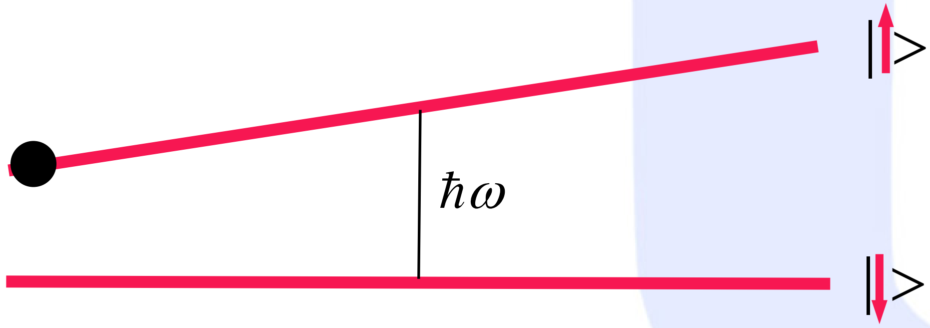
Spin-Spin Coupling





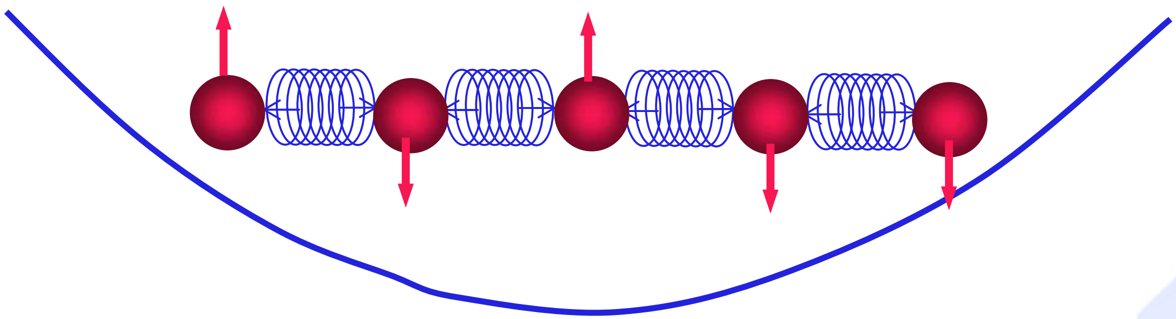
Spin-Spin Coupling





Spin-Spin Coupling

$$\tilde{H} = \frac{1}{2}\hbar\sum_{j=1}^{N}\omega_j(z_{0,j})\sigma_{z,j} + \hbar\sum_{n=1}^{N}\nu_n(a_n^+a_n) - \hbar\sum_{i<j}^{N}\sigma_{z,i}\sigma_{z,j}\left[\frac{1}{2}\sum_{n=1}^{N}\nu_n\kappa_{ni}\kappa_{nj}\right]$$



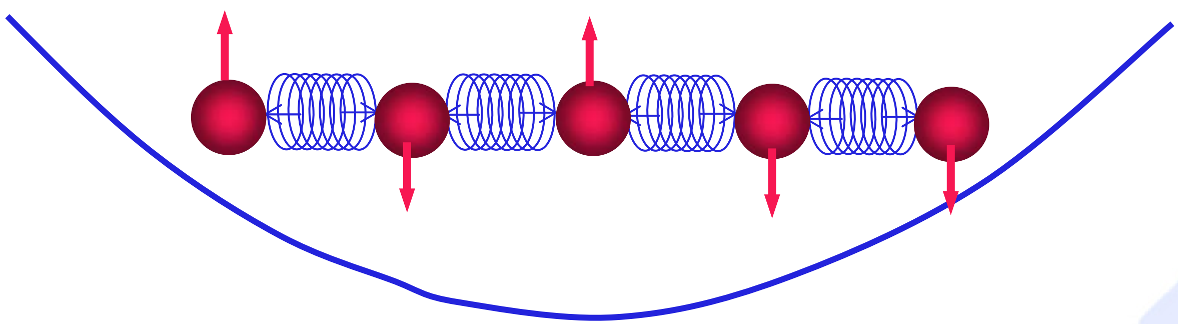
CW in *Laser Physics at the Limit*, Springer, 2002, p. 261.
also: quant-ph/0111158

Ion Molecule

Spin-Spin coupling

$$\tilde{H} = H_{intern} + H_{extern} - \hbar \sum_{i<j}^{N} J_{ij}\, \sigma_{z,i}\sigma_{z,j}$$

$$J_{ij} \propto \left(\frac{\partial_z B}{\nu_1}\right)^2$$



Individual N-qubit “designer molecule”
with adjustable coupling constants

CW in *Laser Physics at the Limit*, Springer, 2002, p. 261.
also: quant-ph/0111158.

D.Mc Hugh, J. Twamley PRA **71**, 012315 (2005), quant-ph/0310015

Spin coupled system using *optical* force instead:
D. Porras and J. I. Cirac PRL **92**, 207901 (2004)

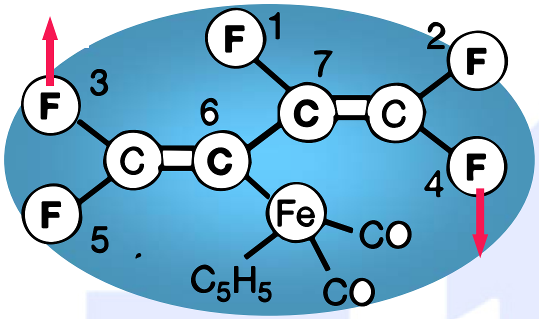
Analogy with NMR

- Intricate quantum algorithms demonstrated.
- Technological basis: coherent manipulation using rf and microwave radiation.

- Macroscopic ensemble
$\Rightarrow$ exponential cost
- Design of molecules nontrivial

Conditional dynamics:

$$\hbar \sum_{i<j}^{N} J_{ij} \; \sigma_{z,i}\sigma_{z,j}$$



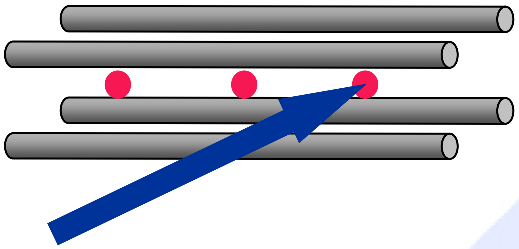
Vandersypen et al., Nature **414**, 883 (2001).

Analogy with NMR

- Intricate quantum algorithms demonstrated.
- Technological basis:
  coherent manipulation using
  rf and microwave radiation.

- Macroscopic ensemble
$\Rightarrow$ exponential cost
- Design of molecules nontrivial

Ion traps:

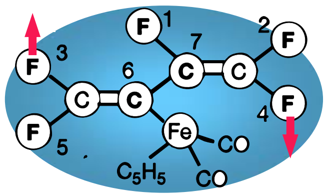


~ Individual qubits.

Use microwaves?

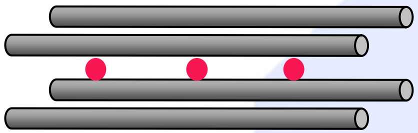
NMR, Trapped Ions, and Ion Molecules

- Coherent manipulation using rf and microwave radiation of long-lived spin states.
- Use sophisticated NMR concepts and techniques.



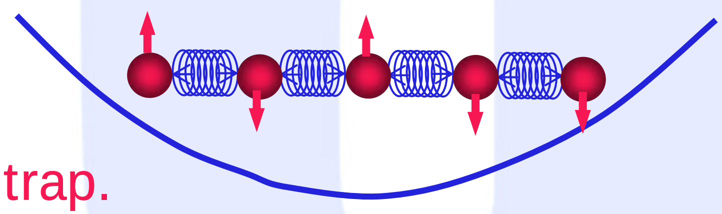
+

- Individual qubits.
- Efficient preparation and readout using projective measurements.



≈

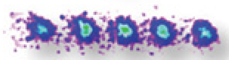
- Spin-spin coupling adjustable.
- (Nearly) insensitive to thermal excitation. ⇒ many ions in single trap.



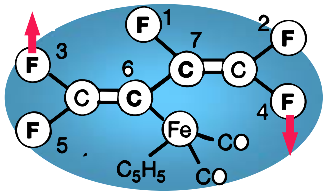
M. Loewen, CW, Verhandl. DPG 2004 (VI) 39, 7/87 (2004)

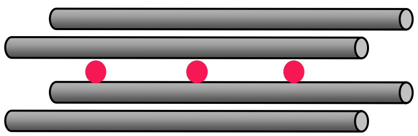
4th European QIPC Workshop, Oxford, 2003 .

NMR, Trapped Ions, and Ion Molecules

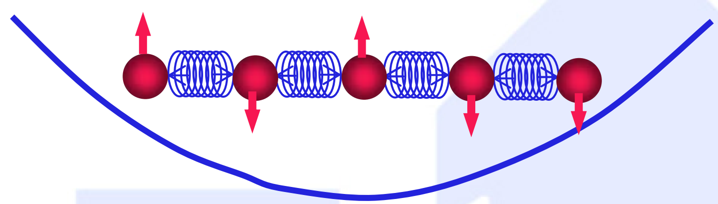


- Coherent manipulation using
  rf and microwave radiation of long-lived spin states.
- Use sophisticated NMR concepts and techniques.
- Individual qubits.
- Efficient preparation and readout using projective measurements.





- Spin-spin coupling adjustable.



many ions in single trap:

- Multi-qubit gates, e.g. Benjamin, NJP **6**, 61 (2004).

- Q.Simulations, e.g. Porras, Cirac, PRL **92**, 207901 (2004).

- Transport of Q.Information, e.g. Christandl et al., PRL **92**, 187902 (2004), Noah, Linden, PRA **69**, 052315 (2004).

- Entanglement and decoherence, e.g. Dür, Briegel, PRL 92, 180403 (2004).

DiVincenzo Criteria for QC

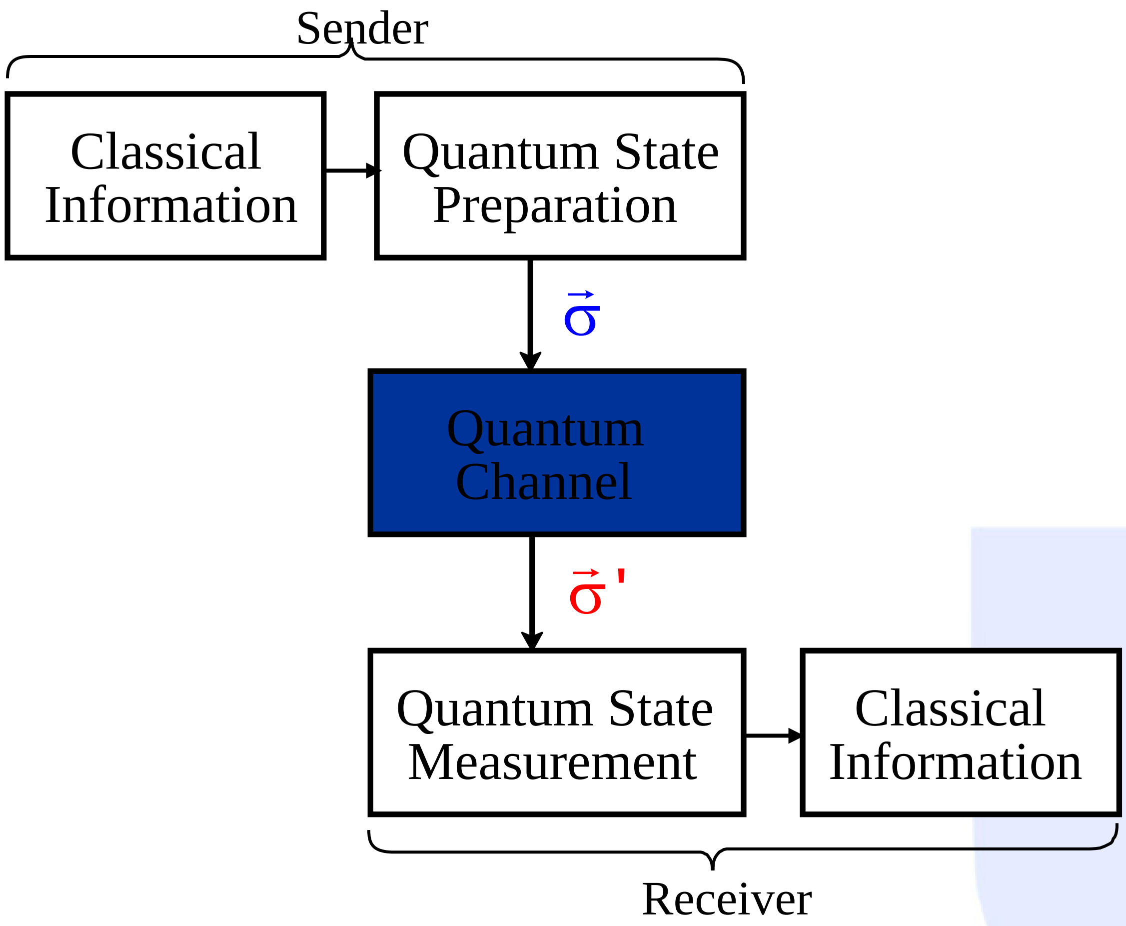
1. *A scalable physical system with well-characterized qubits.*
   Electronic states: qubits; vibrational motion used as bus qubit
   Scalability: schemes in progress.
2. *The ability to initialize the state of the qubits to a simple fiducial state.*
   Individual qubits prepared by effcient optical pumping.
3. *Long decoherence times, much longer than the gate-operation time.*
   Longitudinal relaxation: seconds (electronic) to years (hyperfine);
   transverse relaxation: tens of seconds (hyperfine).
   Gate operation: tens of μs.
4. *A universal set of quantum gates.*
   Single-qubit gates and variety of two-qubit gates experimentally demonstrated.
5. *A qubit-specific measurement capability.*
   Projective measurement with efficiency close to 100% (electron shelving).

D. P. DiVincenzo, Fortschr. Phys.,48 (2000) 771.

Overview

1. Ion Trap and Laser Cooling
   - Electrodynamic trap
   - Collective ion motion: harmonic oscillator
   - Doppler cooling
   - Trapped atom-light interaction
   - Resolved sideband cooling
2. Qubits and Quantum Gates
   - E2-transition, Hyperfine transition
   - Single qubit gates
   - 2-qubit gate
3. Ion Spin Molecules
   - Spin-Motion coupling
   - Spin-Spin coupling
   - Analogy with NMR
4. QIS with trapped Yb^{+} ions
   - Self-learning estimation of quantum states.
   - Quantum process estimation.
   - Quantum Zeno paradox.

Qubit dynamics



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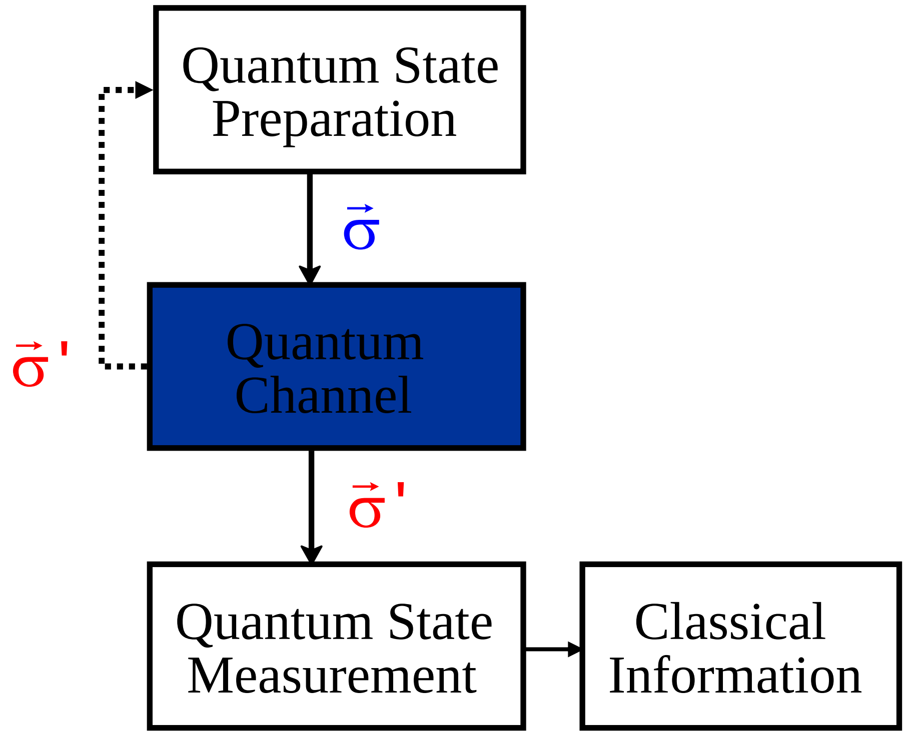
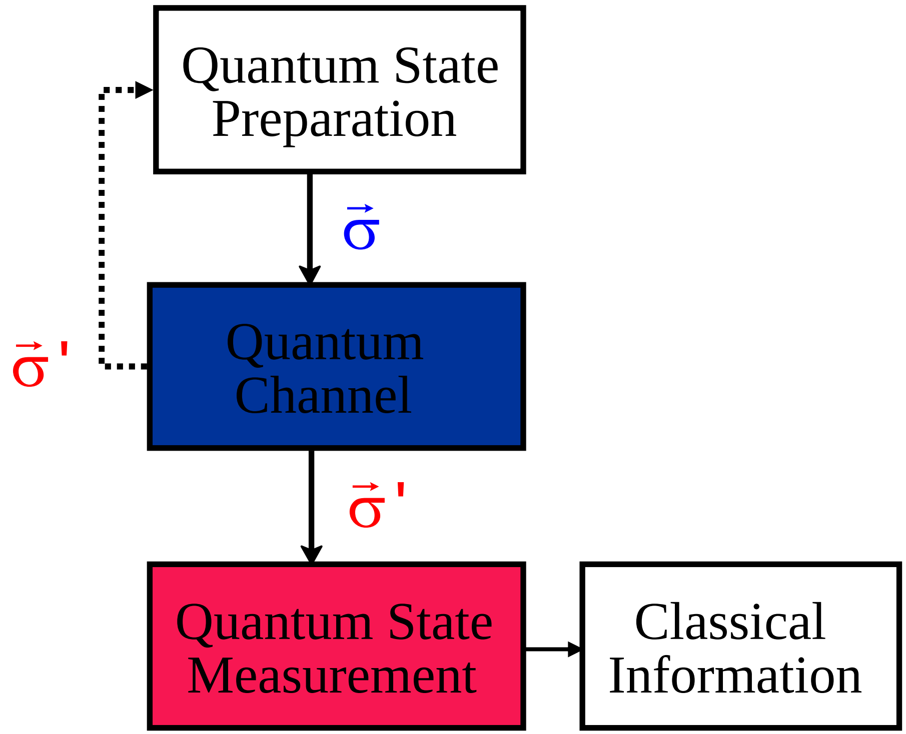
Qubit dynamics



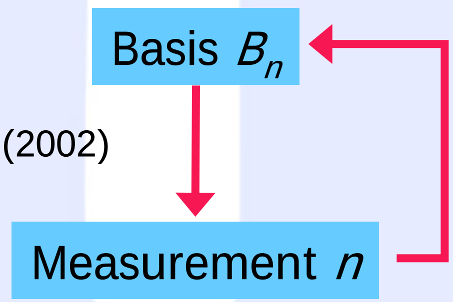
Qubit dynamics



Estimating a quantum state

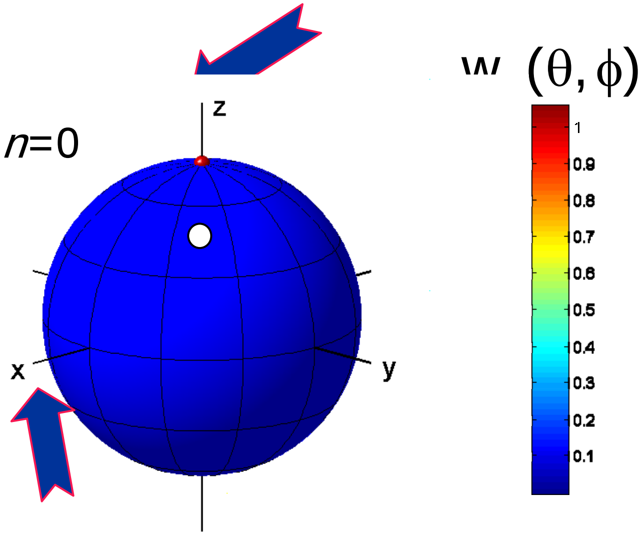
- Estimation using a finite number N of identically prepared qubits:
  - Optimal estimation requires entangled basis
    N=2: A. Peres, W.K. Wootters, PRL **66**, 1119 (1991); S. Massar, S. Popescu, PRL **74**, 1259 (1995). $N \leq 5$: J. I. Latorre, P. Pascual, and R. Tarrach, PRL **81**, 1351 (1998).
  - First experiments (N=2)
    V. Meyer et al., PRL **86**, 5870 (2001).

- Separate (LOCC) measurements on N qubits: Adaptive scheme
  D. G. Fischer, S. H. Kienle, and M. Freyberger, Phys. Rev. A **61**, 032306 (2000).
  E. Bagan, M. Baig, and R. Munoz-Tapia, PRL **89**, 277904 (2002).



- UNOT:
  V. Buzek, M. Hillery, R.F. Werner, PRA **60**, R2626 (1999)
  F. De Martini, V. Buzek, F. Sciarrino, and C. Sias, Nature **419**, 815 (2002)

Adaptive Estimation of a quantum state



- Probability density on Bloch sphere after measurement *n.*

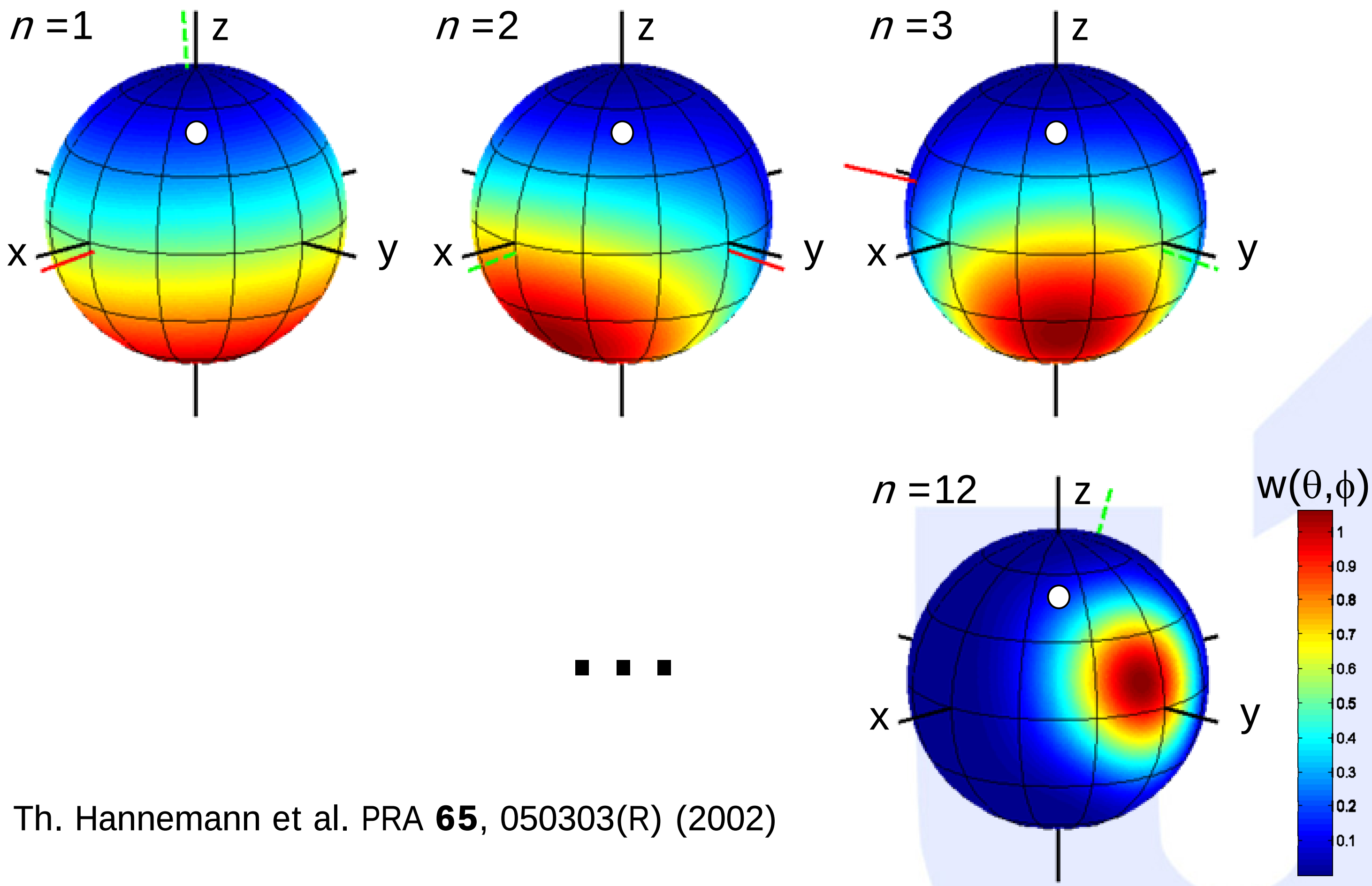
$$\rho_n = \int_0^{\pi} d\theta \sin\theta \int_0^{2\pi} d\phi \, w_n(\theta,\phi) \left|\theta,\phi\right\rangle\left\langle\theta,\phi\right|$$

- Calculate direction of next (*n*+1) measurement from $w_n(\theta,\phi)$ by maximizing expected fidelity

$$F_{n+1}(\theta,\phi) = \left\langle\theta,\phi\right|\rho_{n+1}\left|\theta,\phi\right\rangle$$

Th. Hannemann *et al.* PRA **65**, 050303(R) (2002)

Adaptive Estimation of a quantum state

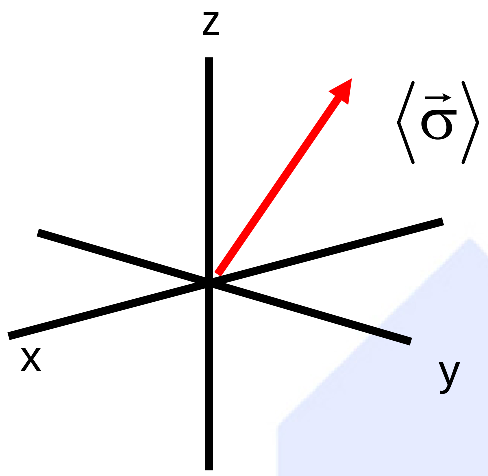


Th. Hannemann et al. PRA **65**, 050303(R) (2002)

Adaptive Estimation of a quantum state

- Account for decoherence:
  - Preparation $\eta_{prep} = 89\ \%$
  - Detection $\eta_{det} = 97\ \%$

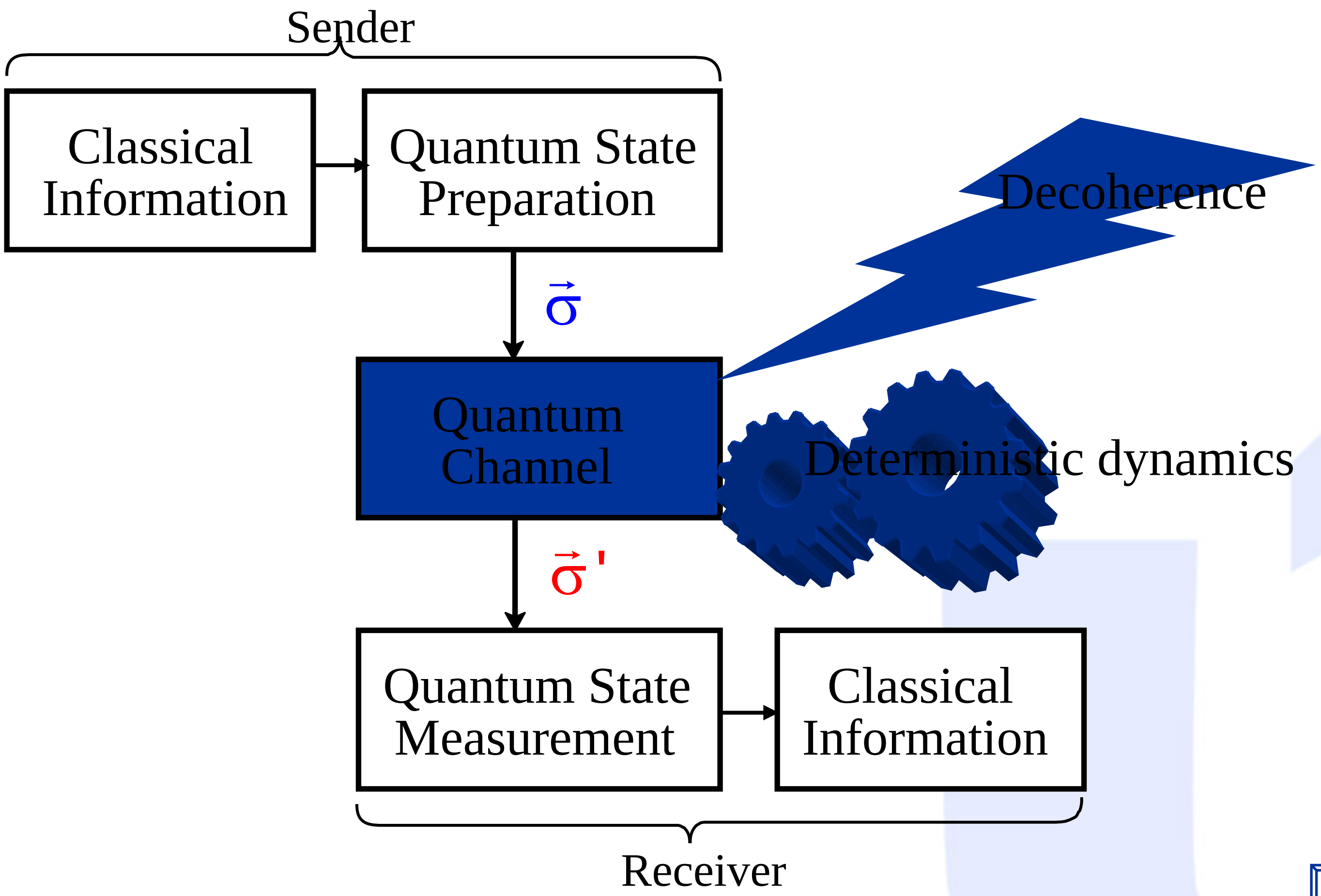
Bloch vector



$\langle F \rangle_{Exp}$	$\lvert \langle \vec{\sigma} \rangle \rvert$	$\langle F \rangle_{Theo} \left(\lvert \langle \vec{\sigma} \rangle \rvert \right)$
(85.0±0.6) %	(74.8±2.1) %	(85.4±0.7) %

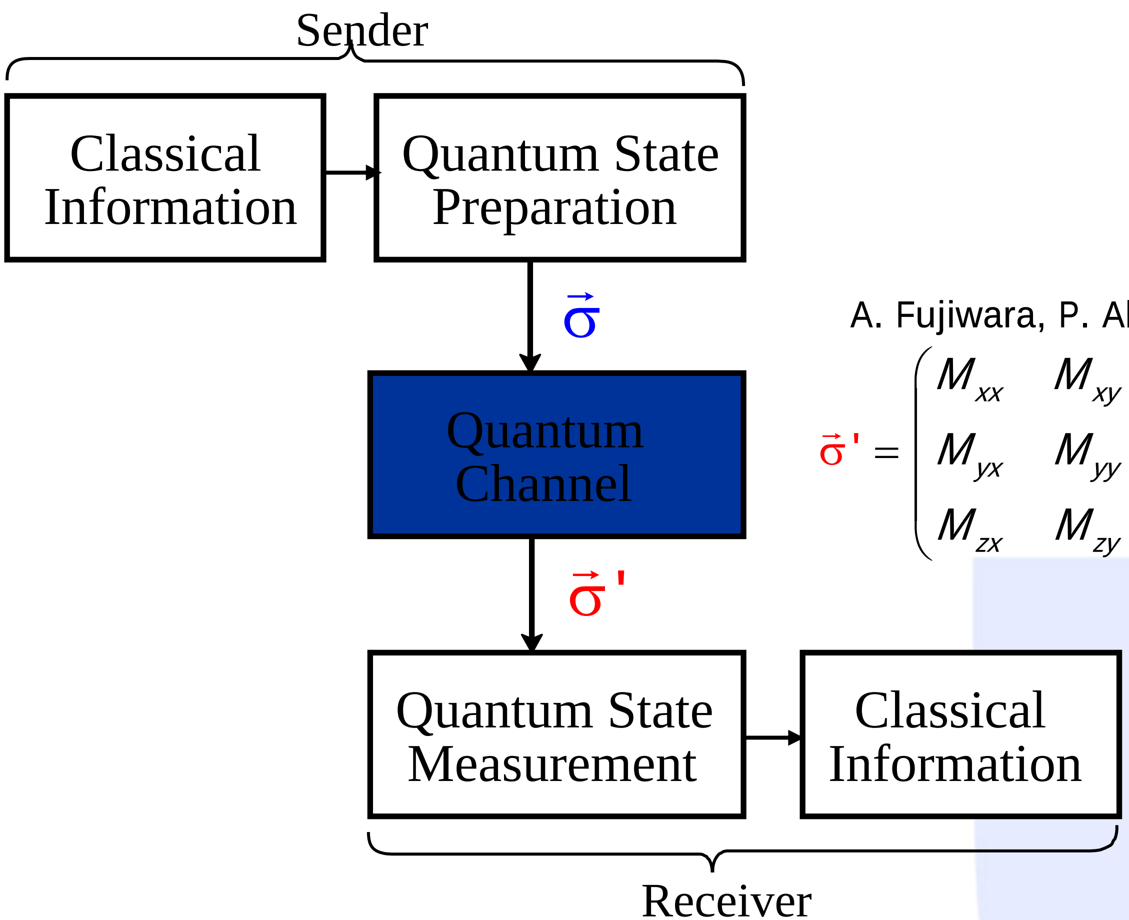
Quantum process estimation

Realization of quantum channels



Quantum process estimation

Realization of quantum channels



A. Fujiwara, P. Algoet, PRA **59**, 3290 (1999):

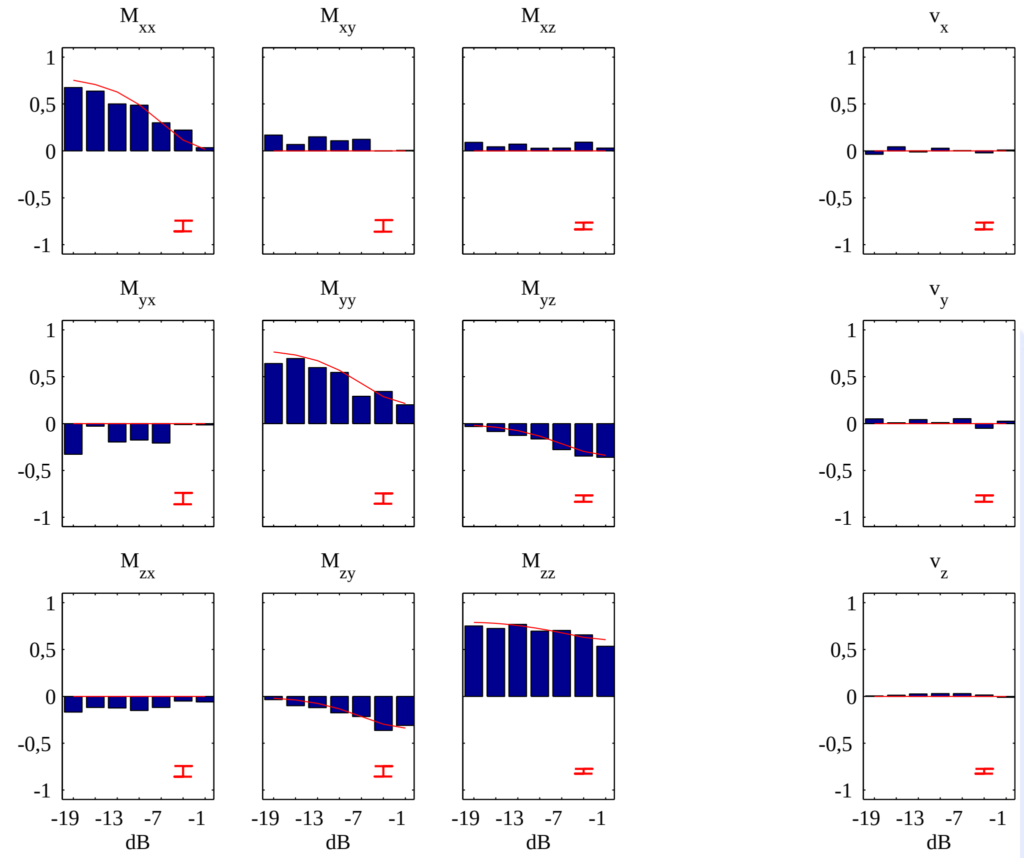
$$\vec{\sigma}\,' = \begin{pmatrix} M_{xx} & M_{xy} & M_{xz} \\ M_{yx} & M_{yy} & M_{yz} \\ M_{zx} & M_{zy} & M_{zz} \end{pmatrix} \cdot \vec{\sigma} + \begin{pmatrix} v_x \\ v_y \\ v_z \end{pmatrix}$$

Quantum process estimation

Realization of quantum channels

Phase damping in arbitrary plane. Here θ=π/6

$$\begin{pmatrix} 1-2\lambda & 0 & 0 \\ 0 & 1-2\lambda\cos^2\theta & -2\lambda\cos\theta\sin\theta \\ 0 & -2\lambda\cos\theta\sin\theta & 1-2\lambda\sin^2\theta \end{pmatrix}$$

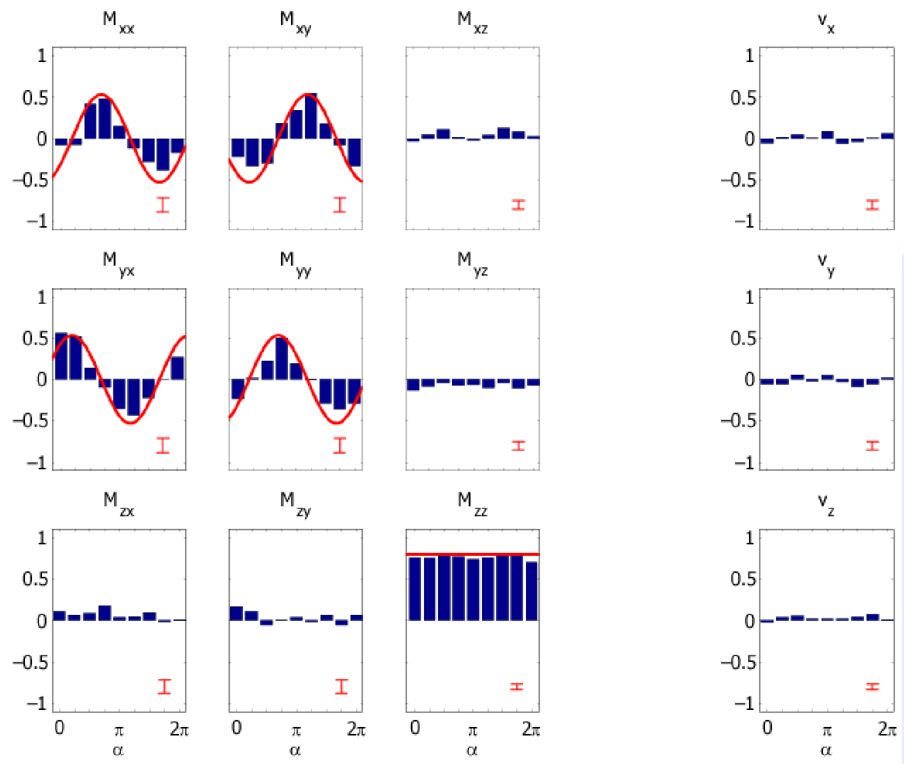


Quantum process estimation

Realization of quantum channels

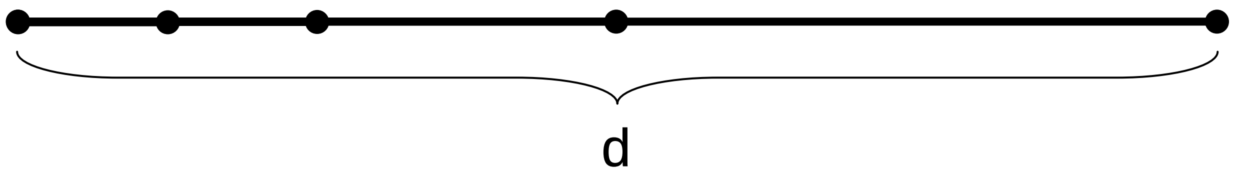
Fixed phase damping and arbitrary polarization rotation

$$\begin{pmatrix} (1-2\lambda)\cos\alpha & (1-2\lambda)\sin\alpha & 0 \\ -(1-2\lambda)\sin\alpha & (1-2\lambda)\cos\alpha & 0 \\ 0 & 0 & 1 \end{pmatrix}$$



Quantum Zeno Paradox
Introduction

Zeno of Elea: “Motion is not possible”

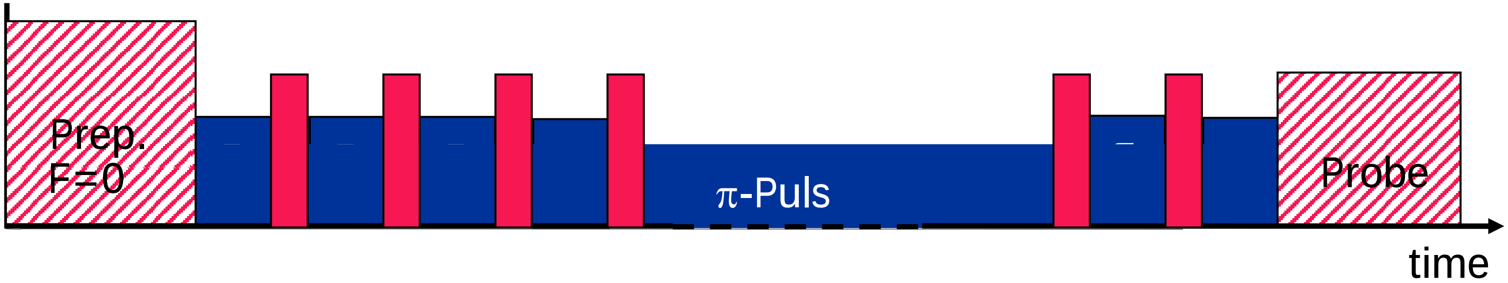


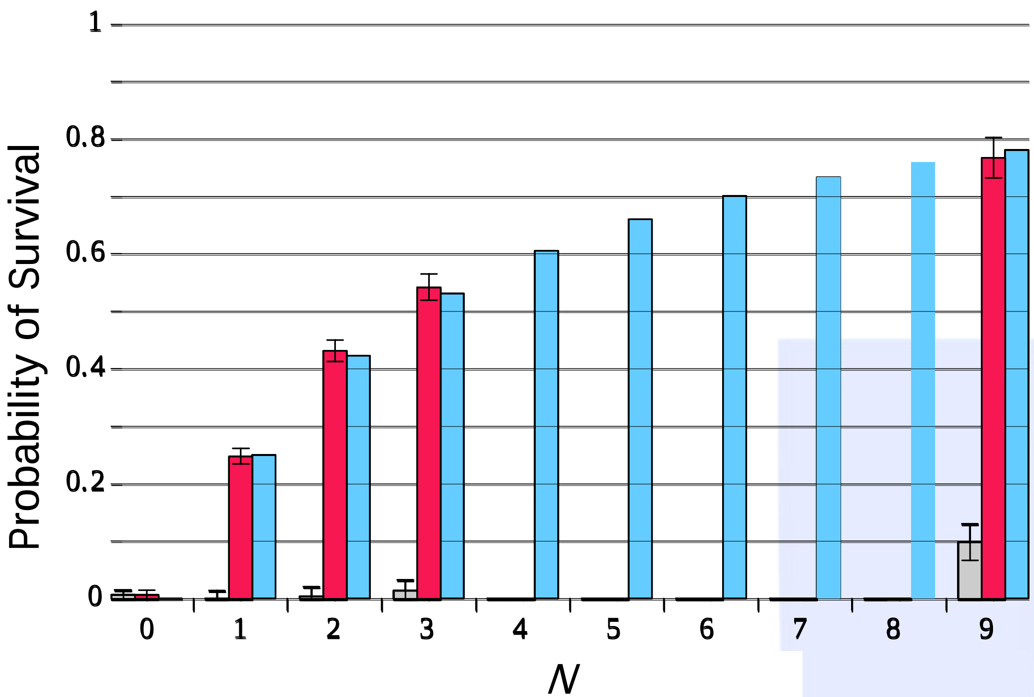
Quantum Mechanics: Slowing down or complete halt of a system’s dynamics under observation.

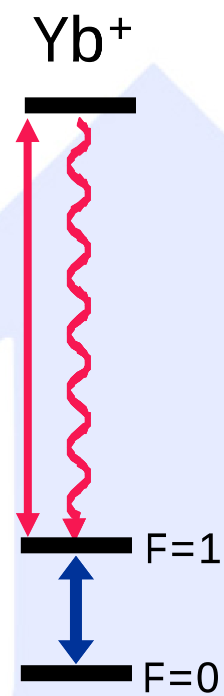
B. Misra, E. C. G. Sudarshan, J. Math. Phys. (NY) 18, 756 (1977)

Quantum Zeno Paradox

Experiment





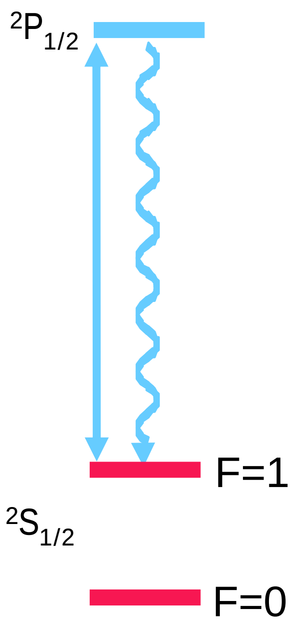


Ch. Balzer *et al.* Opt.Comm. **211**, 235 (2002)

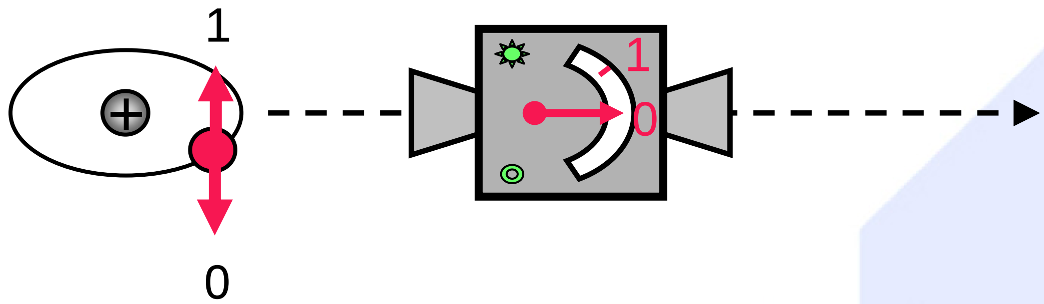
Quantum Zeno Paradox

Experiment

Yb^+



- Correlation measurement apparatus-quantum system.
- Repeated Null-Measurements impede dynamics.



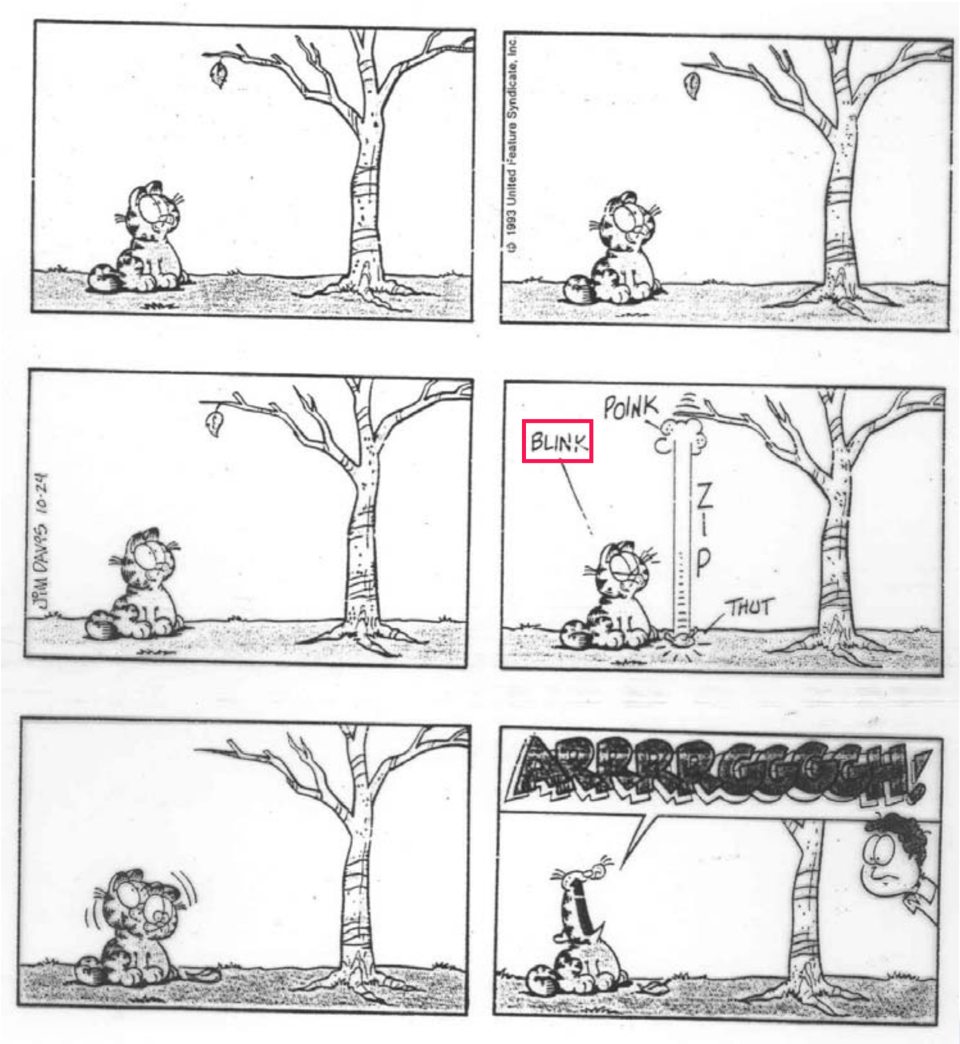
$$\begin{matrix} |1\rangle \otimes |\rightarrow\rangle \\ |0\rangle \otimes |\rightarrow\rangle \end{matrix} \longrightarrow \begin{matrix} |1\rangle \otimes |\nearrow\rangle \\ |0\rangle \otimes |\rightarrow\rangle \end{matrix}$$

D. Home, M. Whitaker, Ann. Phys. N.Y. 258, 237 (1997)

Ch. Wunderlich, Ch. Balzer, Adv. At. Mol. Opt. Phys. **49**, 293 (2003)

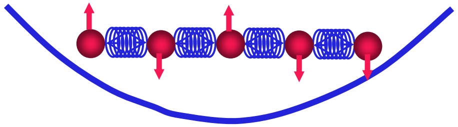
Quantum Zeno Paradox

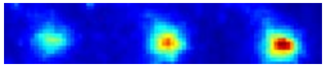
in every day life



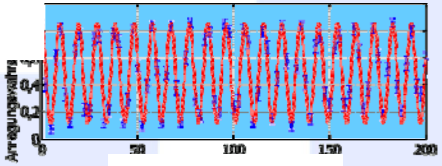
Quantum Optics in Siegen

- Open positions: PhD / Postdoc





Emmy-Noether-Campus



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